64. Scalar Mesons below 1 GeV

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64.1 Introduction

The identification of the light scalar mesons is a long-standing puzzle. Scalar resonances are difficult to resolve experimentally as they can have large decay widths, which cause a strong overlap between neighboring resonances and with background. In addition, in some cases, several decay channels open up within a short mass interval (*e.g.* at the $K\bar{K}$ and $\eta\eta$ thresholds), producing cusps in the line shapes of the nearby resonances. Furthermore, one expects non-$q\bar{q}$ scalar objects, such as hadronic molecules and multiquark states, in the mass range of interest (for reviews see, *e.g.*, Refs. [1–6]).

Light scalars are produced, for example, in πN scattering on polarized/unpolarized targets, $p\bar{p}$ annihilation, central hadronic production, $J/\psi(1S)$, B-, D- and K-meson decays, $\gamma\gamma$ formation, and ϕ radiative decays. Especially for the lightest scalar mesons simple parameterizations like Breit-Wigner functions and variants thereof fail — this is demonstrated explicitly on the example of the $f_0(500)$ or σ, e.g., in Ref. [7]. Accordingly, more advanced theory tools are necessary to extract the resonance parameters from data. In the analyses available in the literature, fundamental properties of the amplitudes such as unitarity, analyticity, Lorentz invariance, and chiral and flavor symmetry are implemented at different levels of rigor. Especially, chiral symmetry implies the appearance of zeros, the so-called Adler zeros, close to the threshold in elastic S-wave scattering amplitudes involving soft (pseudo) Goldstone bosons [8, 9], which may be shifted or removed in associated production processes [10]. Moreover, especially for the lightest non-strange and strange scalar resonance precision extractions of pole parameters get complicated by the presence of both left-hand cuts as well as circular cuts (for a recent review on the subject see Ref. [5]). The methods employed are the K-matrix formalism, the N/D-method, the Dalitz-Tuan ansatz, the inverse amplitude method, unitarized quark models with coupled channels, effective chiral field theories and the linear sigma model, *etc.* Dynamics near the lowest two-body thresholds in some analyses are described by crossed channel (t, u) meson exchange or with an effective range parameterization instead of, or in addition to, resonant features in the s-channel. Dispersion theoretical approaches are applied to pin down the location of resonance poles for the low mass states [11–16] unambigously — in particular the existence of a nonet of light scalar is not questioned anymore.

In parallel to the developments sketched above also lattice QCD entered the field of precision spectroscopy, since it was acknowledged that a study of resonances calls for a proper treatment of the scattering process employing e.g. the Lüscher method [17], later extended to coupled channels in Refs. [18–23]. This allowed for determinations of the poles of the $I = 1/2$ [24], the $I = 1$ [25] as well as the $I = 0$ [26, 27] light scalar mesons. Besides and additional firm confirmation of the existence of a nonet of light scalar mesons, via the quark mass dependence of the pole locations (here the most sophisticated analysis is reported in Ref. [28]) these studies allow for an alternative look on their nature [29–31].

The mass and width of a resonance are found from the position of the nearest pole in the process amplitude (S- or T-matrix) at an unphysical sheet of the complex energy plane, traditionally labeled as

$$\sqrt{s_{\rm Pole}} = M - i\,\Gamma/2\ . \tag{64.1}$$

It is important to note that in general the pole of a Breit-Wigner parameterization does not agree with the S- or T-matrix pole. For a detailed discussion of this issue we refer to the review on

Resonances in this Review of Particle Physics (RPP).

In this review we present proposed values for the mass parameters of the scalar resonances below 1 GeV. Note that those are labeled as 'our estimate' — it is not an average over the quoted analyses, but is chosen to include the bulk of the analyses. An averaging procedure is not justified, since the analyses use overlapping or sometimes even identical data sets so that they are not statistically independent.

In this note, we discuss the light scalars below 1 GeV organized in the *Listings* under the entries $K_0^*(700)$ (or κ) with isospin $I = 1/2$, $a_0(980)$ with $I = 1$, as well as $f_0(500)$ (or σ) and $f_0(980)$ both with $I = 0$. The $I = 2$ $\pi\pi$ and $I = 3/2$ $K\pi$ partial waves do not exhibit resonant behaviour.

64.2 The $K_0^*(700)$, also known as κ ($I = 1/2$)

The $K_0^*(700)$ shows up as a pole in the low energy πK scattering, although its presence and properties are difficult to establish, since it appears to have a very large width ($\Gamma \approx 500$ MeV) and resides close to the $K\pi$ threshold. Hadronic D- and B-meson decays provide additional data points in the vicinity of the $K\pi$ threshold and are discussed in detail in the *Review on Multibody Charm Analyses* in this RPP. With a few exceptions discussed there, the three- or more-hadron final states are usually treated as non-interacting two-body systems. Precise information from

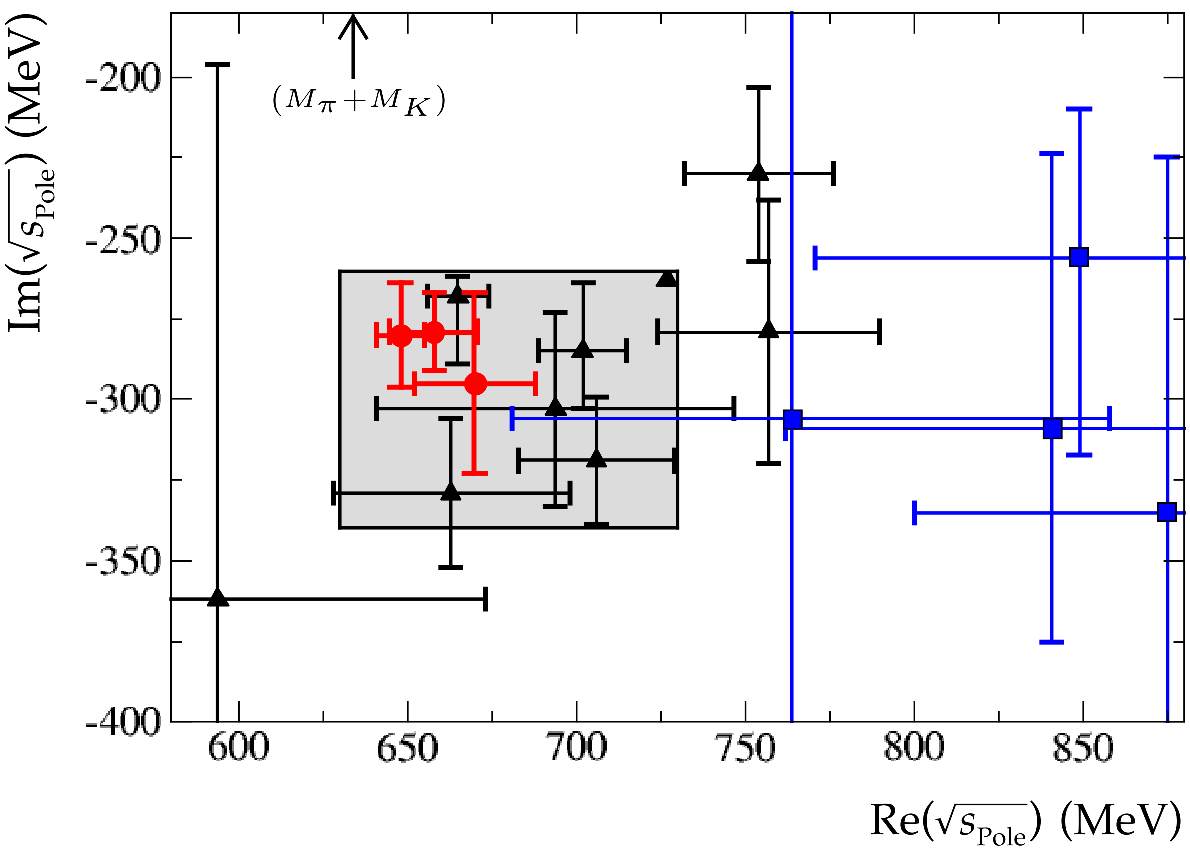

Figure 64.1: Location of the $K_0^*(700)$ (or κ) poles in the complex energy plane. Red circles denote the results of the most sophisticated analyses based on dispersion relations (see text for details), poles extracted from Breit-Wigner fits are shown as blue squares, while all other analyses quoted in the *Listings* are denoted by black triangles. The corresponding references are given in the listing. The arrow shows the location of the πK thresholds. The grey box indicates the range of pole locations classified as 'our estimate' .

semileptonic D decays, where the strongly interacting two-particle final states could be treated without approximation, is not available. BES II [32] (re-analyzed in [33]) finds a $K_0^*(700)$-like structure in $J/\psi(1S)$ decays to $\bar{K}^{*0}(892)K^+\pi^-$ where $K_0^*(700)$ recoils against the $K^*(892)$. The decay $\tau^- \to K_S^0\pi^-\nu_\tau$ can be considered clean with respect to final-state interactions and is studied by Belle [34], with $K_0^*(700)$ parameters fixed to those of Ref. [32].

Some authors find a $K_0^*(700)$ pole in their phenomenological analysis (see, *e.g.*, [35–44]), while others do not need to include it in their fits (see, *e.g.*, [45–48]; note that in Ref. [49] only a K_0^* meson above 825 MeV was discarded using scattering data fits and analyticity, which is not in conflict with the properties quoted below). All works including constraints from chiral symmetry at low energies naturally find a light $K_0^*(700)$ below 800 MeV, see, *e.g.*, [50–56]. The analyses of Ref. [15, 16] are based on the Roy-Steiner equations, which include analyticity and crossing symmetry constraints. Ref. [57] uses the Padé method to extract pole parameters after refitting scattering data constrained to satisfy forward dispersion relations. All three arrive at compatible pole positions for the $K_0^*(700)$ that are also consistent with the pole parameters deduced from other theoretical methods. Due to their large uncertainties, the pole locations deduced from the Breit-Wigner fits appear to be just about consistent with the other determinations, but the real parts of all those analyses lie systematically higher. Moreover, phase shifts extracted from the Breit-Wigner functions for the $K_0^*(700)$ are very different from the known scalar πK phase shifts. The various poles are shown in in Fig. 64.1. The compilation in this figure motivates the pole parameters of the $K_0^*(700)$, which we quote as ‘our estimate’, namely,

$$\sqrt{s^\kappa_{\text{Pole}}} = (630 - 730) - i(260 - 340)\ \text{MeV}\ . \tag{64.2}$$

For an extensive discussion about the πK system in general and the κ meson in particular, see Ref. [58].

64.3 The $a_0(980)$ ($I = 1$)

The $a_0(980)$ couples strongly to the channels $\pi\eta$ and $K\bar{K}$. Independent of any model, the $K\bar{K}$ component must be large in the $a_0(980)$ wave function, since the mass of the $a_0(980)$ lies very close to the opening of the $K\bar{K}$ channel, to which it strongly couples [59, 60]. This generates a pronounced cusp-like behavior in the resonant amplitude and to reveal its true coupling constants the presence of the $K\bar{K}$ channel cannot be ignored. All listed $a_0(980)$ measurements agree on a mass position value near 980 MeV, but the width deduced from the imaginary part of the pole location has values between 50 and 100 MeV, mostly due to the different models. For example, the analysis of the $p\bar{p}$ annihilation data [59] using a unitary K-matrix description finds a width determined from the T-matrix pole of 92 ± 8 MeV. Note that the width of the $a_0(980)$ line shape is typically smaller than what could be expected from the pole location.

The relative coupling $K\bar{K}/\pi\eta$ is determined indirectly from $f_1(1285)$ [61–63] or $\eta(1410)$ decays [64–66], from the line shape observed in the $\pi\eta$ decay mode [67–70], or from the coupled-channel analysis of the $\pi\pi\eta$ and $K\bar{K}\pi$ final states of $p\bar{p}$ annihilation at rest [59].

For the extraction of the $a_0(980)$ pole locations, Refs. [52, 53, 55, 71] use unitarized chiral perturbation theory. Ref. [72] uses a similar formalism to extract the pole of the $a_0(980)$, employing the amplitude fixed in Ref. [73]. A dispersion theoretical approach to the isovector scalar $\pi\eta - K\bar{K}$ system is presented in Ref. [74] that may be refined further from studies of heavy meson decays [75]. Those efforts lead to a rather precise determination of the $a_0(980)$ pole location [76]. A value consistent for the mass parameter, but with a larger width, is found in the analysis of $\bar{p}p$ annihilation in flight data employing a K-matrix [77]. The poles presented in Refs. [55, 59, 70, 76, 77] are shown in Fig. 64.2 together with the range of pole parameters estimated by us from this compilation, namely,

$$\sqrt{s_{\text{Pole}}^{a_0(980)}} = (970 - 1020) - i(30 - 70) \text{ MeV} \tag{64.3}$$

indicated by the box.

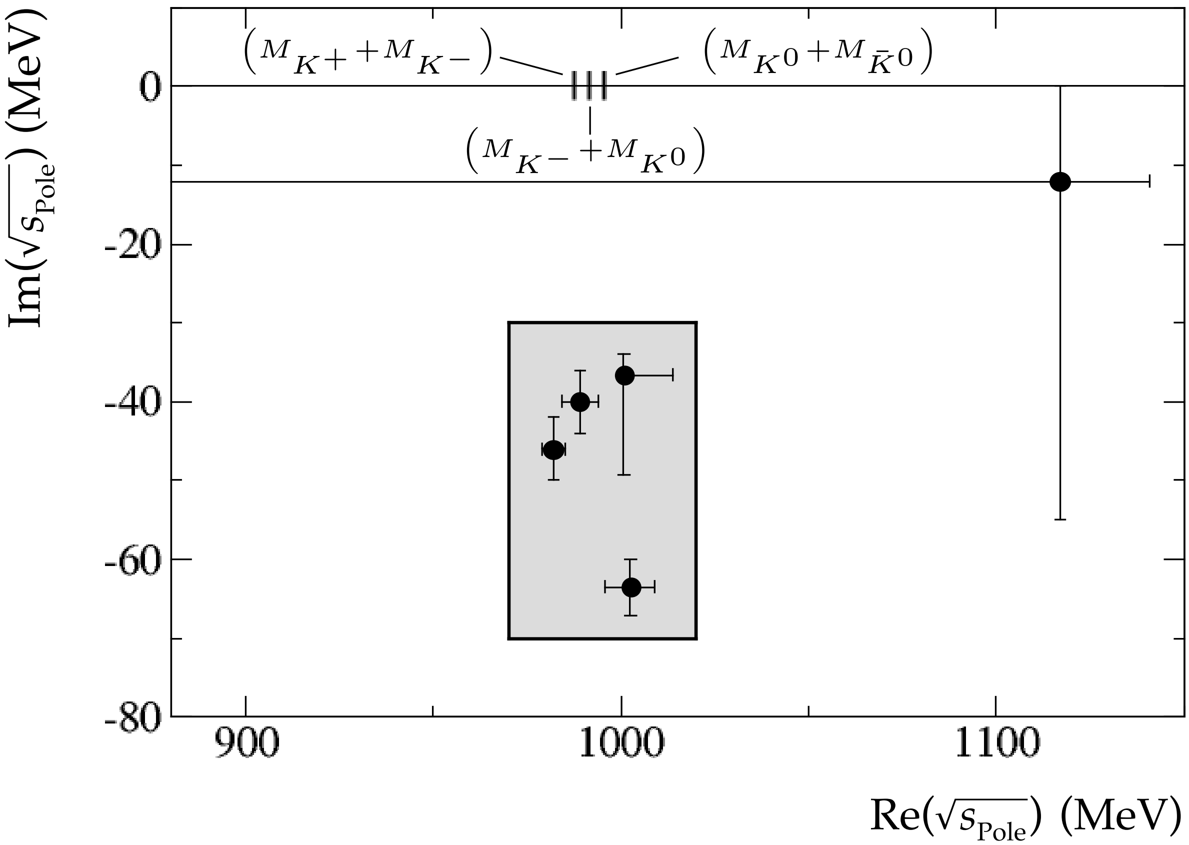

Figure 64.2: Location of the $a_0(980)$ poles from different extractions in the complex energy plane. The corresponding references are given in the *Listings*. Also shown are the thresholds for the K^+K^- and $K^0\bar{K}^0$ channels, relevant for $a_0(980)^0$, and for the K^-K^0 channel, relevant for the $a_0(980)^-$. The grey box indicates the range of pole locations classified as 'our estimate'. Note that by default we take the pole located closest to the physical axis; thus the poles shown are not all on the same sheet.

64.4 The $f_0(500)$, also known as σ-meson ($I = 0$)

For discussions of the $\pi\pi$ S wave below the $K\bar{K}$ threshold and on the long history of the $f_0(500)$, which was suggested in linear sigma models more than 50 years ago, see the review [5]. Information on the $\pi\pi$ S-wave phase shift $\delta^I_J = \delta^0_0$ was already extracted many years ago from πN scattering [78–80], and near the $\pi\pi$ threshold from K_{e4} decays [81]. The kaon decays were later revisited leading to consistent data with very much improved statistics [82, 83]. The reported $\pi\pi \to K\bar{K}$ cross sections [84–87] have large uncertainties. The πN data have been analyzed in combination with high-statistics data (see entries labeled as RVUE for re-analyses of the data). The $2\pi^0$ invariant mass spectra, extracted from $p\bar{p}$ annihilation at rest into $3\pi^0$ [88, 89] and into $5\pi^0$ [90] and from central pp collision [91] do not show a distinct resonance structure below 900 MeV, but these data sets are consistently described with the standard solution for the $\pi\pi$ scalar isoscalar S-wave extracted from high energy $\pi N \to \pi\pi N$ data [79, 92], which allows for the existence of the broad $f_0(500)$. An enhancement is observed in the $\pi^+\pi^-$ invariant mass near threshold in the decays $D^+ \to \pi^+\pi^-\pi^+$ [93–95] and $J/\psi(1S) \to \omega\pi^+\pi^-$ [96, 97], and in $\psi(2S) \to J/\psi(1S)\pi^+\pi^-$ with very limited phase space [98, 99].

The precise $f_0(500)$ (or σ) pole is difficult to establish because of its large width. The $\pi\pi$ scattering amplitude shows an unusual energy dependence due to the presence of the Adler zero in the unphysical regime close to the threshold [8, 9], required by chiral symmetry. However, most of the analyses listed under $f_0(500)$ agree on a pole position near $(500 - i\,250$ MeV). In particular, analyses of $\pi\pi$ data that include unitarity, are consistent with the near threshold $\pi\pi$ data from K_{e4} decays, and the chiral symmetry constraints from Adler zeroes and/or scattering lengths find a light $f_0(500)$, see, *e.g.*, [100, 101].

Precise pole positions with an uncertainty of less than 20 MeV (see our table for the T-matrix pole in the *Listings*) are extracted using the Roy equations, which are twice subtracted dispersion relations derived from crossing symmetry and analyticity. In Ref. [12] the subtraction constants are fixed to the S-wave scattering lengths a_0^0 and a_0^2 derived from matching the Roy equations and two-loop chiral perturbation theory [11]. The only additional relevant input to fix the $f_0(500)$ pole is the $\pi\pi$-wave phase shift at some higher energy point, chosen as 800 MeV. The analysis is improved further in Ref. [14]. Alternatively, in Ref. [13] Roy equations are used as constraints for a fit to data. In that reference also once-subtracted Roy–like equations, called GKPY equations, are used, since the extrapolation into the complex plane based on the twice-subtracted equations leads to larger uncertainties, mainly due to the limited experimental information on the isospin 2 $\pi\pi$ scattering length. Ref. [102] uses Padé approximants for the analytic continuation. All these extractions find consistent results. Using only analyticity and unitarity to describe data from $K_{2\pi}$ and K_{e4} decays, Ref. [103] finds consistent values for the pole position and the scattering length a_0^0. The importance of the $\pi\pi$ scattering data for fixing the $f_0(500)$ pole is nicely illustrated by comparing analyses of $\bar{p}p \to 3\pi^0$ omitting [88, 104] or including [89, 105] information on $\pi\pi$ scattering: while the former analyses find an extremely broad structure above 1 GeV, the latter find $f_0(500)$ masses of the order of 400 MeV.

From Fig. 64.3 we estimate the range of pole positions for the $f_0(500)$, namely,

$$\sqrt{s^{\sigma}_{\rm Pole}} = (400 - 550) - i(200 - 350)\ \text{MeV}\ . \tag{64.4}$$

The plot contains the poles from Refs. [40, 51, 53, 55, 56, 69, 71, 78, 89, 95, 98–101, 103, 105–124] as well as the advanced dispersion analyses [11–14, 102]. The extracted $f_0(500)$ pole position is very sensitive to the high accuracy low energy $\pi\pi$ scattering data [82, 83]. In fact, all analyses consistent with this data find poles within the accepted range indicated in the figure. As in case of the $K_0^*(700)$, poles extracted from Breit-Wigner analyses are shown as blue squares. Again we see that those poles have the tendency to appear at higher masses, although here the effect is not as pronounced as in case of the $K_0^*(700)$. One should not take this as an indication that using Breit-Wigners is justified in case of the $f_0(500)$, since $\pi\pi$ phase shifts extracted from Breit-Wigners are in strong discrepancy with the scattering phase shifts.

If one uses just the most advanced dispersive analyses of Refs. [11–14], shown as solid dots in Fig. 64.3 to determine the pole location of the $f_0(500)$, the range narrows down to [5]

$$\sqrt{s^{\sigma}_{\rm Pole}} = (449^{+22}_{-16}) - i(275 \pm 12)\ \text{MeV}\ , \tag{64.5}$$

which is labeled as 'conservative dispersive estimate' in this reference.

Besides $\pi\pi$, the only other decay channel of the $f_0(500)$ is two photons. Due to the large full width of the $f_0(500)$ an extraction of its two-photon width directly from data is not possible. Thus,

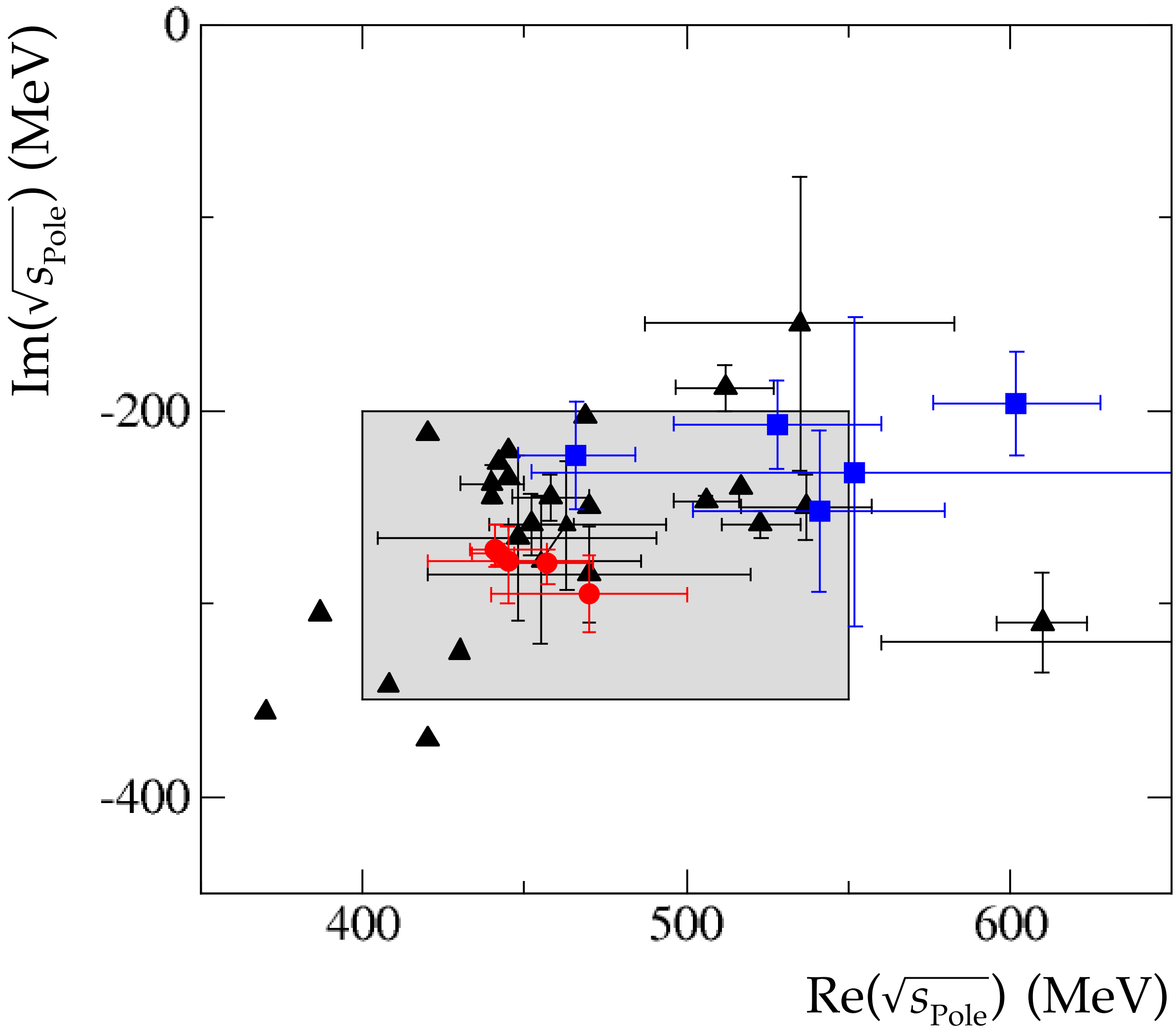

Figure 64.3: Location of the $f_0(500)$ (or σ) poles in the complex energy plane. Circles denote the recent analyses based on Roy(-like) dispersion relations, poles extracted from Breit-Wigner fits are shown as blue squares, while all other analyses are denoted by triangles. The corresponding references are given in the *Listings*. The grey box indicates the range of pole locations classified as 'our estimate' .

the values for $\Gamma(\gamma\gamma)$ quoted in the literature as well as the in *Listings* are based on the expression in the narrow width approximation [125] $\Gamma(\gamma\gamma) \simeq \alpha^2|g_\gamma|^2/(4{\rm Re}(\sqrt{s^\sigma_{\rm Pole}}))$, where g_γ is derived from the residue at the $f_0(500)$ pole to two photons and α denotes the electromagnetic fine-structure constant (see also the review on *Resonances* in this issue of the RPP). The explicit form of the expression may vary between different authors due to different definitions of the coupling constant, however, the expression given for $\Gamma(\gamma\gamma)$ is free of ambiguities. According to Refs. [126–129], $\pi\pi$ and $\gamma\gamma$ scattering data sets are consistently described including $f_0(500)$ via the two step process of $\gamma\gamma \to \pi^+\pi^-$ with pion exchange in the t- and u-channel, followed by a final state interaction $\pi^+\pi^- \to \pi^0\pi^0$. The same conclusion is drawn in Ref. [130], where the $f_0(500) \to \gamma\gamma$ decay width is dominated by re-scattering. Therefore, it might be difficult to learn anything new about the nature of the $f_0(500)$ from its $\gamma\gamma$ coupling. For the most recent work on $\gamma\gamma \to \pi\pi$, see Refs. [56, 71, 103–105, 112–125, 128–133]. There are strong indications (*e.g.*, [134–164]) that the

$f_0(500)$ pole cannot be classified as a $q\bar{q}$ state.

64.5 The $f_0(980)$ $(I = 0)$

The $f_0(980)$ couples predominantly to the $\pi\pi$ and $K\bar{K}$ channels and its signal overlaps strongly with the background represented mainly by the $f_0(500)$ and the $f_0(1370)$. This can lead to a dip in the $\pi\pi$ spectrum at the $K\bar{K}$ threshold. It changes from a dip into a peak structure in the $\pi^0\pi^0$ invariant mass spectrum of the reaction $\pi^- p \to \pi^0\pi^0 n$ [138], with increasing four momentum transfer to the $\pi^0\pi^0$ system, which means increasing the $a_1(1260)$ exchange contribution in the amplitude, while the π exchange decreases. Also when a $(u\bar{u} + d\bar{d})$ source is switched to a $s\bar{s}$ source, as it appears when moving from $B_d \to J/\psi(1S)\pi\pi$ to $B_s \to J/\psi(1S)\pi\pi$, the f_0 signal switches from a dip to a peak [152]. The $f_0(500)$ and the $f_0(980)$ are also observed in the data for radiative ϕ decays ($\phi \to f_0\gamma$) from SND [139, 140], CMD2 [141], and KLOE [142, 143].

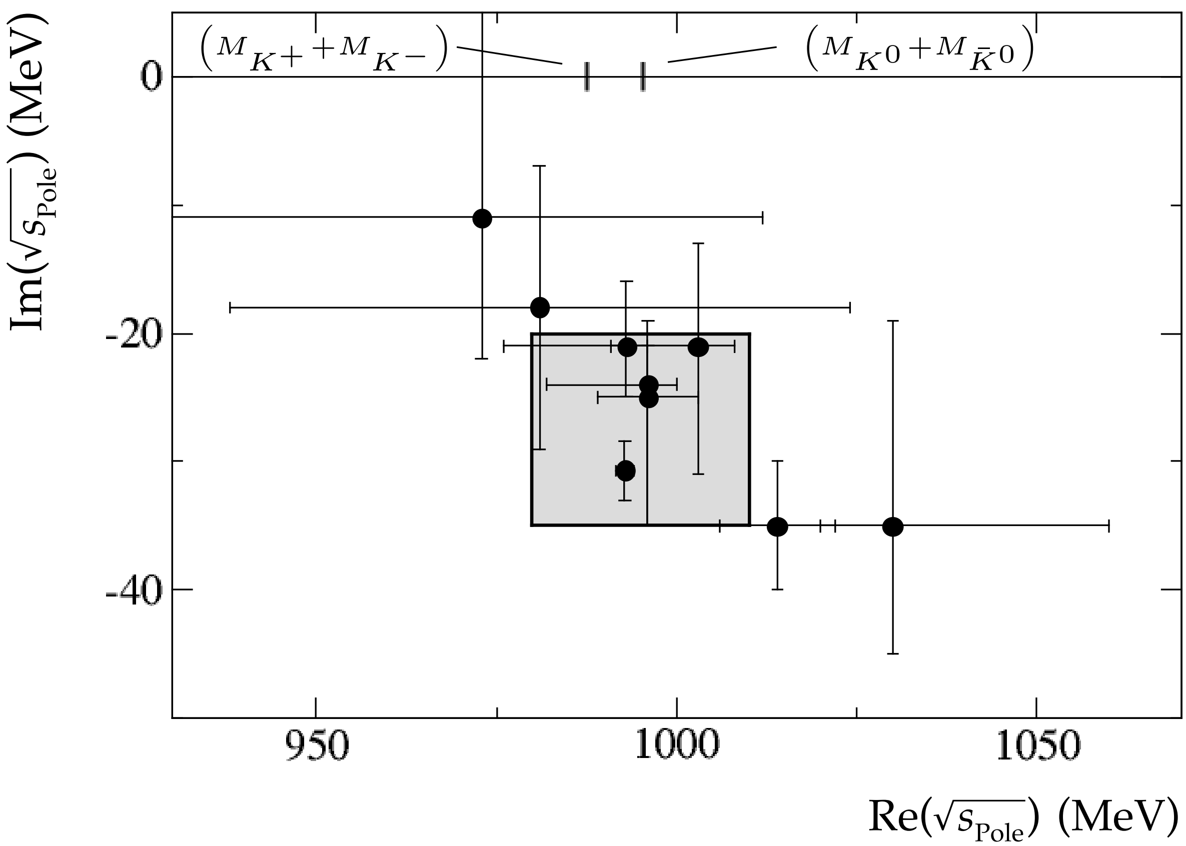

Figure 64.4: Location of the $f_0(980)$ poles from different extractions in the complex energy plane. The corresponding references are given in the *Listings*. Also shown are the thresholds for the K^+K^- and $K^0\bar{K}^0$ channels. The grey box indicates the range of pole locations classified as 'our estimate' .

Unitarized chiral perturbation theory is employed to extract the pole of the $f_0(980)$ in Refs. [52, 53, 55, 71, 72]. Two different dispersive analyses are used in Ref. [13] to simultaneously pin down the pole parameters of both the $f_0(500)$ and the $f_0(980)$. The poles extracted in Refs. [13, 14, 55, 56, 77, 108, 165, 166] are shown in Fig. 64.4, together with the range of pole parameters estimated by us from this compilation, namely,

$$\sqrt{s_{\mathrm{Pole}}^{f_0(980)}} = (980 - 1010) - i(20 - 35)\ \mathrm{MeV} \tag{64.6}$$

indicated by the box. A disclaimer is important here: Both the poles of $a_0(980)$ and of $f_0(980)$ are located very close to the kaon thresholds, with the charged and neutral thresholds being 8 MeV apart — to illustrate this point the pertinent thresholds are shown explicitly in Figs. 64.2 and 64.4. This observation leads to the prediction of an enhanced a_0-f_0 mixing [167–170]. On the other hand, all analyses employed in the pole determinations quoted above are done assuming isospin symmetry. Future studies need to show the impact of isospin violation on the extraction of the $a_0(980)/f_0(980)$ pole parameters.

Analyses of $\gamma\gamma \to \pi\pi$ data [144–146] underline the importance of the $K\bar{K}$ coupling of the $f_0(980)$, while the resulting two-photon width of the $f_0(980)$ cannot be determined precisely [147]. The prominent appearance of the $f_0(980)$ in the semileptonic D_s decays and decays of B and B_s mesons implies a dominant $(\bar{s}s)$ component: those decays occur via weak transitions that alternatively result in $\phi(1020)$ production. Ratios of decay rates of B and/or B_s mesons into $J/\psi(1S)$ plus $f_0(980)$ or $f_0(500)$ are proposed to extract the flavor mixing angle and to probe the tetraquark nature of those mesons within a certain model [148,149]. The resulting phenomenological fits of the LHCb collaboration [150, 151] lead the authors to conclude that their data are incompatible with a model where $f_0(500)$ and $f_0(980)$ are formed from two quarks and two antiquarks (tetraquarks). However, a dispersive analysis of the same data that allows for a model independent inclusion of the hadronic final state interactions in Ref. [152] puts into question the conclusions of Ref. [150]. Moreover, the assumption underlying Ref. [150] that the production dynamics of $f_0(500)$ and $f_0(980)$ are equal such that they cancel in ratios appears to be not justified [5].

64.6 Interpretation of the scalars below 1 GeV

In the literature, many structures are discussed for the light scalar mesons, such as conventional $q\bar{q}$ mesons, compact $(qq)(\bar{q}\bar{q})$ structures (tetraquarks), or meson-meson bound states (hadronic molecules). In reality, there can be superpositions of these components, and one often depends on models to determine the dominant one. Although we have seen progress in recent years, this question remains open. Here, we mention some of the present conclusions.

The $f_0(980)$ and $a_0(980)$ are often interpreted as compact tetraquark states [161–164,171] or $K\bar{K}$ bound states [172]. The insight into their internal structure using two-photon widths [140,173–179] is not conclusive. The $f_0(980)$ appears as a peak structure in D_s decays without $f_0(500)$ background. Based on that observation it is suggested that $f_0(980)$ has a large $s\bar{s}$ component, which according to Ref. [180] is surrounded by a virtual $K\bar{K}$ cloud (see also Ref. [181]). The inclusive production property of the $f_0(980)$ in Z^0 decays is consistent with mesons of other nonets [182]. However, as stated by the authors, the results should be taken as model-dependent measurements while presently no reliable predictions for production rates of non-$q\bar{q}$ states in Z^0 decay are available. Data sets on radiative decays ($\phi \to f_0\gamma$ and $\phi \to a_0\gamma$) from SND, CMD2, and KLOE (see above) are consistent with a prominent role of kaon loops. This observation is interpreted as evidence for a compact four-quark structure of the light scalars in Ref. [183], while it is claimed to point at a molecular nature in Ref. [184,185]. Details of this controversy are given in the comments [186,187]; see also Ref. [188]. There is now a rather broad consensus that the states $f_0(980)$ and $a_0(980)$, together with the $f_0(500)$ and the $K_0^*(700)$, form a nonet of predominantly four-quark states, where at larger distances the quarks recombine into a pair of pseudoscalar mesons creating a meson cloud (see, *e.g.,* Ref. [189]). Different QCD sum rule studies [190–195] do not agree on a tetraquark configuration for the same particle group.

Models that start directly from chiral Lagrangians, either in non-linear [52, 55, 100, 184] or in linear [196–202] realization, predict the existence of the $f_0(500)$ meson near 500 MeV. Here the $f_0(500)$, $a_0(980)$, $f_0(980)$, and $K_0^*(700)$ (in some models the $K_0^*(1430)$) would form a nonet (not necessarily $q\bar{q}$). In the linear sigma models the lightest pseudoscalars appear as their chiral partners.

In the non-linear approaches of Refs. [52] and [100] the above resonances together with the low mass vector states are generated starting from chiral perturbation theory predictions near the first open channel, and then by shifting the states to the resonance locations, using unitarity and analyticity.

Ref. [196] uses a framework with explicit resonances that are unitarized and coupled to the light pseudoscalars in a chirally invariant way. Evidence for a dominant non-$\bar{q}q$ nature of the lightest scalar resonances is derived from their mixing scheme. In Ref. [197] the scheme is extended and applied to the decay $\eta' \to \eta\pi\pi$, which leads to the same conclusions. In Ref. [203] the large N_c behavior of the poles is studied to identify the nature of the resonances generated from scattering equations. This leads to the observation that, while the light vector states behave consistent with what is predicted for $\bar{q}q$ states, the light scalars behave very differently. This finding provides strong support for a dominant non-$\bar{q}q$ nature of the light scalar resonances. Note, the more refined study of Ref. [134] which finds, in the case of the $f_0(500)$, indications for a subdominant $\bar{q}q$ component located around 1 GeV in addition to a dominant non-$\bar{q}q$ nature. Additional support for the dominant non-$q\bar{q}$ nature of the $f_0(500)$ is given in Ref. [204], where the connection between the pole of resonances and their Regge trajectories is analyzed. All works including constraints from chiral symmetry at low energies naturally find a light $K_0^*(700)$ below 800 MeV and a $f_0(500)$ below 600 MeV, see, *e.g.*, [50–55]. In these works the $K_0^*(700)$, $f_0(500)$, $f_0(980)$, and $a_0(980)$ appear to form a nonet [51,54]. Additional evidence for this assignment is presented in Ref. [14], where the couplings of the nine states to $\bar{q}q$ sources are compared. Similar conclusions are reached in Ref. [205] where in the analyzed data the $f_0(980)$ for example appears as a peak structure in $J/\psi(1S) \to \phi\pi^+\pi^-$ while in $J/\psi(1S) \to \omega\pi^+\pi^-$ the signal is suppressed. The same low mass scalar nonet is also found earlier in the unitarized quark model of Ref. [111]. A recent phenomenological study based on a $q\bar{q}$ flavor scheme for the light scalars concludes that their glueball content is ruled out because of low production rates in radiative $J/\psi(1S)$ decays [206].

There are, however, alternative interpretations of the light scalars. For example Ref. [207] (for a more recent, condensed discussion of the idea see Ref. [208]), also builds on chiral symmetry, but expands around an infrared fixed point such that the $f_0(500)$ appears as a QCD dilaton with a mass driven by the QCD scale anomaly. The phenomenology studied in that work appears consistent with this proposal. In Ref. [108,130,209,210] data sets on $\pi\pi - \bar{K}K$ scattering, as well as $\gamma\gamma \to \pi\pi$, are analysed and the authors conclude that especially the $f_0(500)$ should have a significant gluon content. An extensive analysis on mass, width and decay couplings of the light scalars in a QCD spectral sum rule study [211] leads to ambiguous results for their internal structure. The $f_0(500)$ can equally well have tetraquark, molecular or gluonic content, while the states $f_0(980)$ and $a_0(980)$ can be of molecular or tetraquark nature. A scenario where the $f_0(500)$ and $f_0(980)$ result from quarkonia-glueball mixing is also possible. Note, however, that the large similarities of the features of the κ and the σ meson put into question both a dilaton and a glueball contribution to the latter, since neither of them can be present in the former.

A model independent method to identify hadronic molecules goes back to a proposal by Weinberg [212] (an extension of the formalism to virtual states is provided in Ref. [31]), which is shown to be equivalent to the pole counting arguments of [167–170,213–218] in Ref. [216]. The formalism allows one to extract the amount of molecular component in the wave function from the effective coupling constant of a physical state to a nearby continuum channel. It can be applied to near threshold states only and provides strong evidence that the $f_0(980)$ is predominantly a $\bar{K}K$ molecule, while the situation turns out to be less clear for the $a_0(980)$ (see also Refs. [177,179]). This is in line with the findings of Ref. [72], which reports an important role of the $\pi\eta$ channel to the formation of the $a_0(980)$ in addition to the $\bar{K}K$ channel, while the $f_0(980)$ also in this work appears to be predominantly a $\bar{K}K$ molecule. The relevance of both the $\bar{K}K$ and the $\pi\eta$ channels in a dynamically generated $a_0(980)$ is also pointed out in the description of the $\chi_{c1} \to \eta\pi^+\pi^-$ and $D_s^+ \to \pi^+\pi^0\eta$ reactions [219, 220]. A recent analysis on the compositeness of the $\pi\pi - K\bar{K}$ and $\pi\eta - K\bar{K}$ channels for the $f_0(980)$ and $a_0(980)$ resonances based on pole values and decay branchings is performed in Ref. [221]. While the $K\bar{K}$ configuration dominates in the $f_0(980)$, the meson-meson components are subdominant in $a_0(980)$. Further insights into $a_0(980)$ and $f_0(980)$ are expected from their mixing [167–170]. A corresponding signal predicted in Refs. [168, 169] is reported by BES III [222]. The importance of the molecular structure of $a_0(980)$, $f_0(980)$ and of their mixing in the reactions $\bar{B}_s^0 \to J/\psi(1S)\pi^+\pi^-$ and $\bar{B}_s^0 \to J/\psi(1S)\pi^0\eta$ is pointed out in Ref. [223].

In the unitarized quark model with coupled $q\bar{q}$ and meson-meson channels, the light scalars are interpreted as additional manifestations of bare $q\bar{q}$ confinement states, strongly mass shifted from the 1.3 - 1.5 GeV region and very distorted due to the strong ${}^3P_0$ coupling to S-wave two-meson decay channels [218, 224]. Thus, in these models the light scalar nonet comprising the $f_0(500)$, $f_0(980)$, $K_0^*(700)$, and $a_0(980)$, as well as the nonet consisting of the $f_0(1370)$, $f_0(1500)$ (or $f_0(1710)$), $K_0^*(1430)$, and $a_0(1450)$, respectively, are seen as two manifestations of the same bare input states (see also Ref. [225]). It should not remain unmentioned, however, that the heavier nonet lies rather close to the input nonet and that the light scalar one emerges only once the coupling to the two-meson channels is switched on, again highlighting that the meson-meson interaction is indispensable for the light scalars.

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