31. Accelerator Physics of Colliders

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This article provides background for the High-Energy Collider Parameter Tables that follow and some additional information; see in-depth review and a comprehensive list of references in [1]; citations below are limited to widely used textbooks and open access seminal papers and reviews.

31.1 Energy and Luminosity

Collisions of two beams of particles accelerated to high energies $E_{1,2}$ provide access to center-of-mass energies (c.m.e.) of $E_{\rm cm} \approx 2\sqrt{E_1 E_2}$, assuming a typically small or zero crossing angle.

Among the 31 colliders that have ever reached the operational stage (seven remain in operation today), most used particles of equal mass and energy, yielding $E_{\rm cm} = 2E_{\rm b}$. Other machines collide beams of unequal energies—such as electron–proton, electron–ion or proton-ion colliders, or asymmetric B-factories, designed to produce short-lived particles whose decays are more easily detected and analyzed thanks to the resulting Lorentz boost.

In an accelerator, charged particles gain energy from an electric field, which typically oscillates at radio frequency (RF), ranging from hundreds of kHz to tens of GHz. With proper phasing relative to the RF field over a distance l, the energy gain of a particle with charge Ze is proportional to the average accelerating gradient G, i.e., $\Delta E_{\rm b} = ZeGl$. The highest beam accelerating gradients achieved to date in operational machines or dedicated test facilities ($G \approx 100$MV/m in 12GHz normal-conducting (NC) RF cavities and 31.5MV/m in 1.3GHz superconducting (SC) ones) make it possible, in principle, to reach very high energies over reasonably long linear accelerators (linacs), e.g. energies above 100 GeV with a length within 10s of kilometers. However, cost considerations often motivate minimization of RF acceleration via repeated use of the same RF system, which, in that case, would boost the energy in small portions $\Delta E_{\rm b} = ZeV_{\rm RF}$ per turn as the particles repeatedly traverse the total cavity voltage $V_{\rm RF}$. This concept underlies cyclotrons, synchrotrons and storage-ring circular colliders as well as more advanced schemes such as recirculating linear accelerators (RLAs), with or without energy recovery. Circular colliders are by far the most common; in such machines, the momentum and energy of ultra-relativistic particles are determined by the bending radius ρ of the particle trajectory inside the dipole magnets and by the average magnetic field B of these magnets:

$$p = ZeB\rho, \quad E \approx pc \quad \text{or} \quad E_{\rm b}\,[\text{GeV}] = 0.3Z(B\rho)\,[\text{Tm}]\,. \tag{31.1}$$

Such *synchrotron condition* assures approximately constant radius of the beam orbit during acceleration. Transverse focusing by quadrupole magnets is required to confine particles within the narrow aperture of the accelerator beam pipe that passes through the magnets. The maximum field of normal-conducting magnets is about 2 T, limited by the saturation of ferromagnetic materials. While this is sufficient for lower-energy colliders, such as most e^+e^- storage rings, it is inadequate for frontier-energy hadron (or muon) beams, since the implied accelerator tunnels would be excessively long and the total magnet power consumption prohibitively high.

The development of superconducting magnets, employing Nb-Ti wires carrying very high electric current and cooled by liquid helium below 5 K, opened the way to much higher magnetic fields and enabled record-energy hadron colliders [2]. For example, the 14 TeV c.m.e. LHC at CERN uses double-bore SC magnets with a maximum field of 8.3 T and a temperature of 1.9 K, installed in a tunnel of circumference $C \approx 26.7$ km (dipole-magnet bending radius $\rho \approx 2800$ m). The double-bore design permits acceleration of the same particle type in opposite directions, as well as operation with different particle species (e.g., protons and heavy ions) in the two apertures. In contrast, a single-bore design would generally imply counter-propagating beams of particles and antiparticles. Since antiparticle production is energy intensive and therefore limited, the double-aperture concept greatly expands the practical potential of high-performance hadron colliders.

The exploration of rare nuclear and high-energy particle physics phenomena requires not only sufficiently high collision energies but also a large number of detectable events. The expected number of events, N_{exp}, is given by the product of the cross section for the reaction under study, σ_{exp}, and the time integral of the instantaneous *luminosity*, $\mathcal{L}$:

$$N_{\mathrm{exp}} = \sigma_{\mathrm{exp}} \cdot \int \mathcal{L}(t) dt. \tag{31.2}$$

In the Tables, luminosity is stated in the units of $\mathrm{cm}^{-2}\mathrm{s}^{-1}$. The integral on the right is referred to as *integrated luminosity* $\mathcal{L}_{\mathrm{int}}$, and, reflecting the smallness of typical particle-interaction cross-sections is often reported in units of inverse femto- or attobarn, e.g., 1 ab^{-1}=10^{42} cm^{-2}. Colliders usually employ bunched beams of particles with approximately Gaussian distributions, and for two bunches containing N_1 and N_2 particles colliding head-on with frequency f_{coll}, a basic expression for the luminosity is

$$\mathcal{L} = f_{\mathrm{coll}} \frac{N_1 N_2}{4\pi \sigma_x^* \sigma_y^*} \mathcal{F} \tag{31.3}$$

where σ_x^* and σ_y^* characterize the rms transverse beam sizes in the horizontal and vertical directions at the interaction point, and $\mathcal{F}$ is a factor of order 1, that accounts for inefficient geometric overlap of the beams due to a crossing angle, finite bunch length, and dynamic effects such as mutual focusing during the collision (see below). If each beam contains n_{b} bunches, the collision frequency increases to $f_{\mathrm{coll}} = n_{\mathrm{b}} f_0$, where f_0 is either the revolution frequency of a circular collider or the repetition rate of a linear collider. Achieving high luminosity therefore requires maximizing the bunch population and number of bunches, focusing them tightly, and colliding them at high frequencies at dedicated interaction points, where the products of the reactions can be registered by particle detectors.

Subsequent sections in this report briefly expand on the beam dynamics behind collider design, comment on the realization of collider performance in a selection of today's facilities, and end with some remarks on future possibilities.

31.2 Beam Dynamics

Given the enormous and highly concentrated power carried by modern high energy particle beams, the main concern of beam dynamics in colliders is stability of motion of i) individual particles in accelerators, ii) single high-intensity beams of many particles moving together, and iii) colliding beams [3–5].

31.2.1 *Single-Particle Dynamics*

While the reference particle at the nominal energy follows the design trajectory (the reference orbit) primarily determined by the transverse magnetic dipole fields, the other particles in the bunch are confined near it through the focusing action of quadrupole fields. We first focus on the transverse beam dynamics; the longitudinal dynamics—arising from the focusing effect of the RF system, which must remain powered even during steady-energy operation to maintain beam bunching—will be discussed next.

Assume that the reference particle carries a right-handed Cartesian coordinate system, with the co-moving z-coordinate pointed in the direction of motion along the reference trajectory, $z = s - vt$ (with v the reference particle velocity, and t time). The independent variable is the distance s of the reference particle along this trajectory, rather than time t, and for simplicity this reference path is taken to be planar. The transverse coordinates are x (horizontal) and y (vertical), where $\{x, z\}$ defines the plane of the reference trajectory.

Several time scales are involved, and this is reflected in the approximations used in formulating the equations of motion. All of today's high-energy colliders are alternating gradient synchrotrons or, respectively, storage rings and the shortest time scale is set by so-called *betatron oscillations.* The linearized equations of motion of a particle displaced from the reference trajectory are:

$$\begin{gathered} x'' + K_x(s)x = 0 \;, y'' + K_y(s)y = 0 \;,\; z' = -x/\rho(s) \;, \\ \text{with} \;\; K_x \equiv \frac{Ze}{p}\frac{\partial B_y}{\partial x} + \frac{1}{\rho^2} \text{ and } K_y \equiv -\frac{Ze}{p}\frac{\partial B_y}{\partial x} \end{gathered} \tag{31.4}$$

where $\rho = p/ZeB_y$ is the radius of curvature due to the field on the reference orbit. The prime denotes d/ds and the Maxwell equation in vacuum $\nabla \times \mathbf{B} = \mathbf{0}$ helps to eliminate $B_x(s)$ using the relation $\partial B_x/\partial y = \partial B_y/\partial x$. In this linear approximation, the vertical magnetic field $B_y(s)$ in the (x, z)-plane contains only dipole and quadrupole terms, which are treated as static in time, but s-dependent.

The solutions of the Hill's equations (31.4) for x and y with a restoring force periodic in s are those of quasi-harmonic oscillators:

$$x(s) = \sqrt{2J_x\beta_x}\; \cos\psi_x \;,\;\; x'(s) = -\sqrt{\frac{2J_x}{\beta_x}}\,[\alpha_x \cos\psi_x + \sin\psi_x] \;, \tag{31.5}$$

where the *action* J_x is a constant of integration, $\alpha_x = \alpha_x(s) \equiv -(1/2)d\beta_x(s)/ds$, and the envelope of oscillations is modulated by the *beta-function* $\beta_x(s)$. A solution of the same form describes the motion in y. The betatron oscillation phase advances according to $d\psi_x/ds = 1/\beta_x$; that is, $2\pi\beta_x$ also plays the role of a local wavelength of oscillations along the orbit. An extremely important parameter for circular machines is the *tune*, Q_x, which is the number of such oscillations per turn about the closed path:

$$Q_x = \frac{1}{2\pi}\oint d\psi_x = \frac{1}{2\pi}\oint \frac{ds}{\beta_x(s)}. \tag{31.6}$$

While the integer part of the tune $[Q_{x,y}]$ generally characterizes the extent of the focusing lattice, it is the fractional part of the tune $\{Q_{x,y}\}$ that needs to be well defined and controlled by the machine operators in order to stay away from potentially detrimental resonances, which may occur under conditions of $kQ_x + lQ_y = m$, where k, l, and m are integers. For example, for the LHC a combination of horizontal and vertical tunes — also called the *working point* — equal to (Q_x, Q_y)=(64.31, 59.32) has been selected, such that resonances up to the order of $|k| + |l| = 10$ or 12 are avoided. These resonances are driven by high-order multipole components of the fields in the magnets, or by self-fields of the beam, or by the electromagnetic fields of the opposite bunch. Normally, the nonlinear components are very weak compared to linear ones, nevertheless, when the nonlinear resonance condition is encountered, the amplitudes of particle oscillations could grow over the beam lifetime, resulting in the escape of the particles to the machine aperture, in the increase of the average beam size, or in both; either of these is highly undesirable phenomena. Careful analysis of nonlinear beam dynamics is instrumental in determining and optimizing the *dynamic aperture*, which is defined as the maximum amplitude of a bounded particle motion.

Neglecting for now all nonlinear effects and usually small $x - y$ coupling, and considering only the linear dynamics, the beta-function is well defined and satisfies the following equation:

$$2\beta_x\beta_x'' - \beta_x'^2 + 4\beta_x^2 K_x = 4\,. \tag{31.7}$$

In a region free of magnetic fields, such as in the neighborhood of a collider interaction point (IP), usually occupied by particle detectors, a symmetric solution of Eq. (31.7) is a parabola:

$$\beta_x(s) = \beta_x^* + \frac{s^2}{\beta_x^*}\,, \tag{31.8}$$

where, in this case, s denotes the longitudinal distance from the IP. The location of the beam waist usually coincides with the IP and corresponds to the minimum value of the beta-function β_x^*; the asterisk is used to indicate IP parameters.

Note that individual quadrupole magnet focuses particles in one plane and defocuses in the orthogonal one, see Eq.(31.4), and a standard way to provide focusing in both planes is to employ an alternating gradient periodic focusing lattice, consisting of a sequence of equally-spaced quadrupoles with a magnetic field gradient equal in magnitude, but alternating in sign ("focusing quadrupole - drift space - defocusing quadrupole - drift space" – known as a *FODO cell*). Eq. (31.7) has stable periodic solutions $\beta_x(s), \beta_y(s)$ in both planes provided that the focal length of the quadrupoles is longer than half the focusing-lens spacing L, i.e., $f = p/(eB_2 l) > L/2$ (where l is the length of a quadrupole magnet, here assumed to be short $l \ll L$, and $B_2 \equiv |\partial B_y/\partial x|$ the quadrupoles' field gradient). In that case, the beta-functions have maxima at the focusing quadrupoles and minima at the defocusing ones, equal to, for example, $\beta_{\text{max,min}} = (2 \pm \sqrt{2})L$ in the case of $f = L/\sqrt{2}$, which corresponds to a betatron phase advance $\varDelta\psi_{x,y} = 90^\circ$ per FODO cell.

Expressing the invariant J_x in terms of x, x' yields

$$J_x = \frac{1}{2}\left(\gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2\right) = \frac{x^2 + (\alpha_x x + \beta_x x')^2}{2\beta_x} \tag{31.9}$$

with $\gamma_x = \gamma_x(s) \equiv (1 + \alpha_x^2(s))/\beta_x(s)$. In a periodic system, these *Courant-Snyder parameters* [6] (frequently referred to as *Twiss parameters*) $\alpha(s)$, $\beta(s)$, $\gamma(s)$ are usually defined by the focusing lattice; in a single pass system such as a linac, the parameters may be selected to match the x-x' distribution of the input beam. For a given position s in the ring, the transverse particle motion in $\{x, x' \equiv dx/ds\}$ phase space describes an ellipse, the area of which is $2\pi J_x$, where the horizontal action J_x is a constant of motion and independent of s. If the interior of that ellipse is populated by an ensemble of non-interacting and non-radiating particles, that area, given the name *emittance*, is constant over the trajectory as well and would only change with energy. In a typical case of the particle's energy change rate being much slower than betatron motion, and considering a Hamiltonian system (i.e., a hadron collider or a linear collider, either without significant synchrotron radiation), the adiabatic invariant $\int p_x dx$ is conserved, and given that for small angles $p_x = x' \cdot \beta\gamma mc^2$, it is common practice to consider an energy-independent *normalized beam emittance* that is equal to the product of the emittance and relativistic factor $\beta\gamma/\pi$ and denoted by ε_{n}. For a beam with a Gaussian distribution in $\{x, x'\}$, average action value $\langle J_x\rangle$ and standard deviations σ_x, and $\sigma_{x'}$, the definition of the normalized rms beam emittance is

$$\varepsilon_{\text{n}x} \equiv \beta\gamma\langle J_x\rangle = \beta\gamma\frac{\sigma_x^2(s)}{\beta_x(s)} = \beta\gamma\frac{\sigma_{x'}^2(s)}{\gamma_x(s)}\,, \tag{31.10}$$

with a corresponding expression for the other transverse direction, y. The angular brackets denote an average over the beam distribution. For 1D Gaussian beam, 95% of the particles are contained within $\{x, x'\}$ phase space area of $6\pi\varepsilon_{\text{n}}/(\beta\gamma)$. Normalized beam emittances are conserved over the acceleration cycle in linear, static focusing lattices $K_{x,y}(s)$, and consequently, one would expect the same ε_{n} at the hadron (or linear) collider top energy as the one coming from the very initial low energy particle source. Unfortunately, that is rarely the case as many time-varying or nonlinear phenomena come into play. In an e^-/e^+ storage ring, the normalized emittance is not preserved during acceleration, but at each energy the beam's equilibrium emittance is determined by the effect of synchrotron radiation as a balance between radiation damping and quantum excitation [7]. In such a ring, for a given accelerator optics, the normalized equilibrium emittance increases with the third power of the beam energy [8].

As for the description of a particle's longitudinal motion, one takes the fractional momentum deviation $\Delta p/p$ from that of the reference particle as the variable conjugate to z. The factors $K_{x,y}$ and ρ in Hill's equations (31.4) are dependent on momentum p, leading to a number of effects: first, the trajectory of off-momentum particles deviates by $\Delta x(s) = D_x(s)(\Delta p/p)$, where the *dispersion function* $D_x(s)$ is determined by the magnetic lattice and is usually positive, periodic, and of the order of $\sim \rho/Q_x^2$. Second, the radius of curvature and orbit path-length C vary with the momentum and, to first order, are characterized by the momentum compaction factor α_c,

$$\alpha_\mathrm{c} \equiv \frac{\Delta C/C}{\Delta p/p} = \frac{1}{C}\oint \frac{D_x(s)}{\rho(s)} ds\,, \tag{31.11}$$

which typically is of order $1/Q_x^2$. Energy deviations also result in changes of machine focusing lattice properties and variations of the particle tunes, characterized by the *chromaticity* $Q'_{x,y} \equiv \Delta Q_{x,y}/(\Delta p/p)$. The natural chromaticity due to momentum dependence of the quadrupole focusing is negative and large $\sim -Q_{x,y}$. Corresponding chromatic tune variations can, therefore, become unacceptably large even for relatively small energy deviations $(\Delta p/p) \sim (0.1-1)\cdot 10^{-3}$. To assure transverse particle stability, usually, the chromaticity is partially or fully compensated by additional sextupole magnets placed at locations of non-zero dispersion.

Radiofrequency electric fields in s direction provide a longitudinal focusing effect, allowing a stable increase of particle energy. The frequency f_s of such longitudinal *synchrotron oscillations* is (expressed in units of revolution frequency f_0, to become the synchrotron tune Q_s)

$$Q_s \equiv \frac{f_s}{f_0} = \sqrt{\frac{(\alpha_\mathrm{c} - 1/\gamma^2) hZeV_\mathrm{RF}\sin\phi_s}{2\pi\beta c p}}\,, \tag{31.12}$$

where $h = f_{RF}/f_0$ denotes the harmonic number, V_RF the RF voltage, and $\phi_s = \cos^{-1}(\Delta E/ZeV_\mathrm{RF})$ the synchronous phase, with ΔE the average energy loss per turn (e.g. due to synchrotron radiation and impedance). The synchrotron tune Q_s determines the amplitude of longitudinal oscillations for a particle with an initial momentum offset, e.g., the rms bunch length σ_z relates to the rms momentum spread $\delta p/p$ as:

$$\sigma_z = \frac{c(\alpha_\mathrm{c} - 1/\gamma^2)}{2\pi Q_s f_0}\left(\frac{\delta p}{p}\right)\,. \tag{31.13}$$

Similarly to the case of transverse oscillations, the area of the longitudinal phase space $\{\Delta E, \Delta t\}$, or $\{\gamma\beta\delta p/p = (1/\beta)\Delta\gamma, z = \beta c\Delta t\}$, encircled by a moving particle is an adiabatic invariant, and the corresponding normalized *longitudinal emittance* $\varepsilon_{\mathrm{n},L} = \beta\gamma m c\sigma_z(\delta p/p)$ is a generally conserved quantity in hadron accelerators and also in linear accelerators. In the case of lepton storage rings, synchrotron radiation determines the equilibrium relative momentum spread, which grows linearly with beam energy [7, 8], and the corresponding bunch length follows from Eq. (31.13). In hadron synchrotrons, the longitudinal emittance sometimes is intentionally blown up during acceleration, so as to preserve longitudinal beam stability.

Longitudinal oscillations are the slowest of all the periodic processes which take place in the accelerators. For example, in the LHC, the frequency of synchrotron oscillations at the top energy of 7 TeV is about $f_s \approx 23$ Hz, the revolution frequency is $f_0 \approx$11.3 kHz, the frequency of betatron oscillations is about $Q_{x,y} f_{\mathrm{rev}} \approx 700$ kHz and the RF frequency is $f_{\mathrm{RF}} = 400.8$ MHz ($h = 35640$). It should be noted that longitudinal motion is practically absent in linacs. In the absence of bending dipoles, dispersion $D_x(s)$ is zero and so are the momentum compaction factor α_c and the synchrotron tune Q_s. As a result, ultrarelativistic particles in a linac barely change their relative positions during acceleration despite significant energy spread.

Highest-energy circular colliders face a serious impediment in the form of synchrotron radiation (SR) that causes an energy loss per turn of

$$\mathit{\Delta} E_{\mathrm{SR}} = \frac{1}{3\varepsilon_0} \frac{Z^2 e^2 \beta^3 \gamma^4}{\rho}\,, \tag{31.14}$$

here, ε_0 is the permittivity of vacuum. For electrons and positrons, the SR energy loss of the particles per turn can be expressed numerically as $\mathit{\Delta} E_{\mathrm{SR}} = 88.5$ [keV/turn] E_{b}^4 [GeV]$/\rho$ [m]. Such losses must be fully compensated by a correspondingly high total RF voltage per turn. At beam energies above a few hundred GeV, the SR loss per turn of the particles becomes comparable to their energy ($\mathit{\Delta} E_{\mathrm{SR}} \sim E_{\mathrm{b}}$), which makes circular e^+e^- colliders impractical for center-of-mass energies above roughly 500 GeV.

Dynamics of the particle spin and sophisticated methods to maintain beam polarization along the acceleration chain, from the polarized sources to collisions, dedicated spin matching procedures to enable self polarization in e^+/e^- storage rings and the *resonant depolarization* method of ultra-precise c.m.e. calibration are described in [9].

31.2.2 *High Intensity Beams*

Ultimate collider luminosity calls for high beam currents $I_{\mathrm{b}} = Zef_0 n_{\mathrm{b}} N$, where N is the intensity of a single bunch. Three related major difficulties include growing RF demands to compensate the synchrotron-radiation power loss $P = I_{\mathrm{b}} \mathit{\Delta} E_{\mathrm{SR}}$ in e^+/e^- beams, the advent of so-called *coherent (*or *collective) beam instabilities*, and growing demands for minimization of radiation due to inevitable particle losses. Many types of single- and multi-bunch instabilities are caused by beam interactions with electromagnetic fields induced by the beam itself due to the *impedance* of the vacuum chambers and RF cavities [4], or caused by unstable clouds of secondary particles, like electrons or ions, which are formed around the circulating beams [10]. These instabilities can develop as quickly as within tens to thousands of turns and need to be controlled. Mechanisms that are routinely employed to avoid coherent instabilities include the use of nonlinear magnets to generate sufficient spread of the tunes and therefore, provide *Landau damping*, fast beam-based transverse and longitudinal feedback systems, and electron/ion clearing (either by weak magnetic or electric fields or by modulation of the primary beam current profile rendering secondaries unstable, or by reducing the yield of secondary electrons via either a special coating or extensive *beam scrubbing* of the vacuum chamber walls). Careful control of the accelerator impedance during the design phase is another critical and routinely applied strategy for mitigating these instabilities.

High current beam operation is sensitive to even minuscule fractional intensity losses caused by particles' scattering at a large angle or with a large energy loss, sufficient for either the particle amplitudes $\sqrt{2J_{x,y}\beta_{x,y}(s)}$, or their dispersive position deviations $\mathit{\Delta} x = D_x(s)(\mathit{\Delta} p/p)$ to exceed the available transverse aperture, usually set by collimators (otherwise, by the vacuum chamber and magnet apertures). This can be due to residual vacuum molecules near the beam orbit or Compton scattering off thermal photons, due to Coulomb scattering off other particles within the same bunch (*Touschek effect*), or due to collisions with opposite beam particles and fields, such as inelastic interaction of protons, Bhabha scattering $e^+e^- \to e^+e^-$, or radiative Bhabha scattering $e^+e^- \to e^+e^-\gamma$ (see corresponding chapters in [11]).

Particles can also get lost on the aperture as a result of much slower mechanisms of diffusion caused either by the above processes with smaller scattering amplitudes, but stochastically repeated many times, such as intensity-dependent multiple Coulomb *intrabeam scattering* [12], by external noises such as ground motion or magnetic field fluctuations, or via chaotic mechanisms like *Arnold diffusion*, modulational diffusion, or resonance streaming in nonlinear fields, enhanced by minor tune modulations. Diffusion leads to a slow evolution of the beam distribution function and appearance of highly unwanted large-amplitude tails and beam emittance growth. The only way to counteract it is to arrange *beam cooling* (damping of particle oscillations). The cooling requires a reaction force opposite to particle momentum arranged such that, on average, the corresponding dissipative particle energy loss is compensated for by external power [13, 14].

In the case of electron or positron storage rings, such cooling occurs naturally due to synchrotron radiation and provides an automatic route to achieve small equilibrium emittances through a balance between radiation damping and excitation of oscillations by random radiation of individual photons. Fast radiation damping allows *top-up injection* of new particles without removing existing ones, a useful method to maximize the integrated luminosity of circular e^+e^- colliders. Synchrotron radiation damping will also be an important cooling mechanism for future energy-frontier hadron colliders, like the proposed FCC-hh and SPPC (see below). Four other methods of beam cooling have been developed and successfully employed to attain low emittances, namely *electron cooling* and *stochastic cooling* of heavy particles (ions and antiprotons), *laser cooling* of ion beams, and the *ionization cooling* of muons.

To prevent damage or excessive irradiation of accelerator components—ensuring they remain accessible for maintenance in the tunnel—sophisticated collimation systems are employed. These systems typically consist of a series of targets or primary collimators that scatter halo particles, followed by numerous absorbers (sometimes as many as a hundred) positioned at dedicated locations to intercept stray particles [15, Ch.9.7]. In the highest-energy modern and proposed future colliders, extreme total beam energies, $n_\mathrm{b}NE_\mathrm{b}$, ranging from megajoules to gigajoules, and transverse energy densities reaching several GJ/mm^2 present one of the most significant challenges for achieving high-efficiency and robust particle collimation.

31.2.3 *High Luminosity Collisions*

Eq. (31.3) for luminosity can be recast in terms of normalized transverse emittances Eq. (31.10) and the beta-functions β^* at the IP as:

$$\mathcal{L} = f_0 \gamma n_\mathrm{b} \frac{N^2}{4\pi\sqrt{\varepsilon_{\mathrm{n}x}\beta_x^*\varepsilon_{\mathrm{n}y}\beta_y^*}}\mathcal{F}\,. \tag{31.15}$$

Here, equal bunch populations N are assumed in two Gaussian beams with identical emittances. To achieve high luminosity, one must maximize the total beam population $n_\mathrm{b}N$ while minimizing the emittances, and collide the beams at high frequency at locations where the focusing optics provides the lowest possible amplitude functions $\beta_{x,y}^*$. Achieving this requires sophisticated strong-focusing systems - the so-called *low-beta insertions* - sometimes occupying a significant fraction of the collider's total length. The minimum $\beta_{x,y}^*$ is limited by the maximum field gradients and apertures of the interaction region (IR) magnets, as well as by the effectiveness of chromatic and nonlinear aberration compensation.

The typical geometric reduction factor is $\mathcal{F} \approx 1$, it seldom drops below 0.5 for most colliders, except in cases dictated by specific physics processes under study. One source of reduction is the *hourglass effect*, arising from the growth of transverse beam sizes as the distance from the IP increases, where $\beta(s)$ expands parabolically as in Eq. (31.8). For long, round bunches, this effect can be approximated as $\mathcal{F} \approx \sqrt{\pi}A\exp(A^2)\mathrm{erfc}(A)$, with $A = \beta^{/}\sigma_z$. Nonzero beam crossing angles, θ_c, for example in the horizontal plane, and long bunches with rms length σ_z, further reduce the luminosity by a factor $\mathcal{F} \approx 1/(1+\Phi^2)^{1/2}$, where $\Phi = \sigma_z \tan(\theta_{\rm c}/2)/\sigma_x^*$ is the so-called *Piwinski angle*.

One of the most common limits to producing high luminosity arises from electric and magnetic forces of the opposite bunch at the IPs, characterized by a dimensionless *beam-beam parameter* :

$$\xi_{x,y} = \frac{r_0 N \beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}^*(\sigma_x^* + \sigma_y^*)} \, , \tag{31.16}$$

where $r_0 = Z^2e^2/(4\pi\varepsilon_0 mc^2)$ is the classical radius of the colliding particle (with charge Ze and mass m). From Eqs. (31.3) or (31.15) and (31.16), one can note that the path to higher luminosity via higher beam intensity and smaller beam sizes almost automatically calls for a higher beam-beam parameter as $\mathcal{L} \propto \xi$. Cited in the Tables, the beam-beam parameter is roughly equal to the betatron tune shift experienced by small-amplitude particles – positive in the case of opposite charge beams, like e^+e^-, and negative for same charge beams as in pp collisions. Beam-beam forces can lead to coherent effects, such as unstable beam oscillations or blow-up of one beam's size while the other beam remains small or even shrinks (*flip-flop effect*). The tune spread arising from ξ and the nonlinear nature of beam-beam interactions results in strong diffusion along high-order transverse resonances $kQ_x + lQ_y = m$ and, ultimately, in beam size growth and/or beam losses. Operational experience indicates that the aforementioned effects are tolerable below certain *beam-beam limit* of $\xi_{x,y} \approx 0.003 - 0.012$ in hadron colliders [16], and – due to strong synchrotron radiation damping – an order of magnitude higher one in e^+e^- colliders, with maximum $\xi_{x,y} \approx 0.03 - 0.12$ [17,18]. The accessible beam-beam parameter range can also be restricted by coherent beam-beam instabilities. These various limits translate into a maximum allowed single bunch intensity N and call for an increase of the number of bunches n_b to achieve higher luminosities.

In linear colliders, where each bunch collides only once, with typically much smaller beam size and experiencing much stronger forces, the strength of the collision is measured by the ratio of the rms bunch length σ_z to the beam-beam focal length. This ratio, called *disruption parameter* D_y, is related to ξ_y via $D_y = 4\pi\sigma_z\xi_y/\beta_y^*$. Stronger beam-beam focusing and higher D_y lead to effectively smaller beam sizes and a resulting luminosity enhancement; but it also makes the collisions more sensitive to small offsets, resulting in a *kink instability*. Additional beam-beam effects arising in the collisions at linear colliders are the emission of *beamstrahlung* (synchrotron radiation in the field of the opposing beam), along with e^+e^- pair creation, and depolarization by various mechanisms.

Beamstrahlung is relevant for both linear colliders, where it may significantly degrade the luminosity spectrum, and for future highest-energy circular colliders, where it may limit the beam lifetime, and also increases the energy spread and bunch length of the stored beam. For both types of colliders, the beamstrahlung is mitigated by making the colliding beams as flat as possible at the interaction point ($\sigma_x^* \gg \sigma_y^*$) to lower the beams' EM fields. The photon energy spectrum of the beamstrahlung is characterized by the parameter $\Upsilon = (2/3)\hbar\omega_{\rm c}/E_{\rm b}$ [19], with $\hbar\omega_{\rm c}$ denoting the critical photon energy. The spectrum strongly deviates from the classical synchrotron radiation spectrum for Υ approaching or exceeding 1.

For hadron colliders, two fundamental luminosity limits are the beam lifetime, determined by burn-off in the collisions due to inelastic pp interaction $dN/dt = -\mathcal{L}\sigma_{\rm in}$, and the radiation from the collision debris, which may induce "quenches" (transitions to the normal-conducting state) of the superconducting final quadrupole magnets, and, in the long term, affect the equipment lifetime. Another limit on the achievable integrated luminosity in circular colliders is set by the minimum or average turnaround time (the time between the beam abort at the end of a physics fill and the start of the next physics collisions). Achieving practical filling times with many bunches in the collider requires either fast cycling injector machines and/or the top-up injection operation. The latter makes the average luminosity of circular electron-positron colliders approximately equal to the peak luminosity.

31.3 Recent High Energy Colliders

In this and the following section, elaboration is made on various issues associated with some of the recently operating colliders, particularly factors which impact integrated luminosity. Only general references are provided, where further information can be obtained. A more complete list of recent colliders and their parameters can be found in the High-Energy Collider Parameters tables.

31.3.1 *Tevatron*

The first superconducting synchrotron in history, the Tevatron [20], was converted into a proton-antiproton collider in 1985. Its 4.4 T dipole magnets employed Nb-Ti superconducting cable operating at 4.5 K, requiring what was then the world's largest cryogenic system. With $\sqrt{s}$ up to 1.96 TeV, it remained the highest-energy collider for 25 years and delivered more than 12 fb^{-1} of integrated luminosity to each $p\bar{p}$ detector experiment (CDF and D0) before being shut down in 2011. The route to high integrated luminosity in the Tevatron was governed by the antiproton production rate, the turnaround time to produce another store, and the resulting optimization of store duration [21]. The antiproton production complex consisted of three 8 GeV $\bar{p}$ accelerators (the Accumulator, Debuncher, and Recycler—the latter being the first high-energy accelerator built with permanent magnets) and employed 25 independent stochastic cooling systems along with a pioneering high-energy electron cooling setup, enabling accumulation of up to a record $25{\cdot}10^{10}$ $\bar{p}$ per hour. Despite severe parasitic long-range interactions between the two beams—each consisting of 36 bunches placed on helical orbits by two dozen ±150 kV high-voltage (HV) separators—a total beam-beam tune shift parameter of $n_{\mathrm{IP}}\xi \approx 0.025$–$0.03$ was achieved, a record for hadron beams, with $n_{\mathrm{IP}} = 2$ primary collision points. Other notable advances included sophisticated longitudinal beam manipulation techniques, such as *slip-stacking* and *momentum mining*, as well as the first operational use of *electron lenses* for beam collimation and for compensation of long-range beam-beam effects [22]. Ultimately, the Tevatron achieved luminosities a factor of 430 higher than its original design specification.

31.3.2 *HERA*

The first lepton–proton collider, the 6.4 km long Hadron–Elektron–Ring–Anlage (HERA) at DESY in Germany [23], operated between 1992 and 2007 and delivered nearly 1 fb^{-1} of integrated luminosity at $\sqrt{s} \approx 320$ GeV to the electron–proton collider experiments H1 and ZEUS [24, Ch.10.5]. HERA was the first facility to employ both major applications of superconductivity: 5 T magnets in the 920 GeV proton ring, and superconducting RF accelerating structures providing approximately 12 MW of power to compensate for synchrotron radiation losses of the 30 GeV lepton beams (electrons or positrons in a conventional-magnet ring). With precise orbit and optics control, the HERA lepton beam naturally became transversely polarized to about 60% within roughly 40 minutes via the *Sokolov–Ternov effect*. Special *spin rotator* magnets were installed on either side of the collider interaction points to provide 30–45% longitudinal polarization at the experiments

31.3.3 *LEP*

Installed in a 26.7 km circumference tunnel, LEP [25] was the largest circular e^+e^- collider constructed to date. It operated from 1989 to 2000 with beam energies ranging from 45.6 to 104.5 GeV. The synchrotron radiation loss per turn reached roughly 3% of the beam energy, making the total available RF voltage and power the limiting factors for LEP's maximum energy and luminosity of 10^{32} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$. At a beam energy of 98 GeV, LEP ran with a beam-beam parameter $\xi \approx 0.083$, corresponding to a total beam-beam tune shift for four interaction points of $n_{\mathrm{IP}}\xi \approx 0.33$. In its final year of operation, 288 SRF cavities were powered by 36 klystrons, each delivering an average of 0.6

MW, providing a total $V_{\mathrm{RF}} = 3.63$ GV. For beam energies up to about 60 GeV, LEP employed the *resonant depolarization* technique to measure the beam energy with 0.001% accuracy.

31.3.4 *SLC*

Based on an existing 3-km long 2.85 GHz warm RF linac, the SLC [26] was the first and only linear collider. It was operated from 1987 to 1998 with a constant beam energy of 45.6 GeV, up to about 80% electron-beam polarization, quasi-flat beams, a final-focus optics with local chromatic correction based on four interleaved sextupoles and $\beta_y^* \approx 1$ mm. In its last year, SLC achieved a peak luminosity of about 3×10^{30} cm^{-2}s^{-1}, roughly half of the design value. The SLC had a high-efficiency positron source providing 5×10^{12} e^+ per s for 120 Hz injection into the linac. It also employed the *BNS damping* to suppress the single-bunch beam break up instability, and also demonstrated an about 2-fold increase of luminosity from *disruption enhancement* due to the mutual focusing of the colliding electron and positron bunches at the interaction point,

31.4 Presently Operating Colliders

31.4.1 *LHC*

With a beam energy of 6.8 TeV (to be raised to the design value of 7 TeV through consolidation and magnet training), the superconducting Large Hadron Collider [27] presently is the world's highest energy collider. In the latest runs, peak luminosities of up to 2.6×10^{34} cm^{-2}s^{-1} have been achieved at 13.6 TeV c.m.e. - more than twice the design value (the current status is best checked at the Web site [28]). To meet its luminosity goals, the LHC operates with a high beam current of approximately 0.5 A, leading to stored energies of about 330 MJ per beam. Controlled energy deposition and component protection are given a high priority and a sophisticated highly efficient system of more than 100 beam collimators is employed [29]. At the energy of about 7 TeV per particle, synchrotron radiation poses a challenge, as the cryogenic system must remove roughly 7 kW due to synchrotron radiation, intercepted with a specially designed *beamscreen* inside the vacuum chamber, at a temperature of about 5–20 K, to be compared with a temperature of 1.9 K for the magnet cold bore. The elevated temperature allows for a more energy-efficient removal of beam-induced heat. The beamscreen also provides an effective cryo-pump for the vacuum system. When synchrotron-radiation photons hit the beamscreen, they can generate photoelectrons. These photoelectrons, and also any other electrons generated in the vacuum system, e.g. by residual-gas ionization, are accelerated in the electric field of the beam and may multiply via secondary-electron emission, with consequent electron cloud development. To mitigate this issue, the beamscreen is regularly subjected to beam-induced surface conditioning (*scrubbing*), thereby lowering the secondary emission yield. The two proton beams of 2556 bunches spaced by 25 ns are contained in separate pipes throughout most of the circumference and are brought together into a single 130 m long beam pipe at the interaction points. To avoid approximately 30 head-on collisions a small crossing angle of about 0.3 mrad is employed, which reduces the luminosity by about 15%. Still, the bunches moving in one direction experience multiple long-range encounters with the counter-rotating bunches and the resulting perturbations of the particle motion substantially contribute to the beam lifetime reduction. The dominant source of approximately 8 hour characteristic luminosity decay time is proton burn-off due to inelastic pp interaction with $\sigma_{\mathrm{in}} \approx 81$ mb, corresponding to *pile-up* of up to 50 (number of events per individual bunch crossing). In special physics runs with a few bunches and large β^*, the LHC achieved a head-on beam-beam tune shift of $n_{\mathrm{IP}}|\xi| \approx 0.02$ with $n_{\mathrm{IP}} = 2$, about twice as high as in regular operation.

The Tables also show the LHC luminosity performance in Pb-Pb collisions, which for the ATLAS and CMS experiments well exceeded the design value, while for the ALICE experiment, the luminosity is *levelled* near the Pb-Pb design value of 10^{27} cm^{-2}s^{-1}. The LHC can also provide Pb-p collisions as it did in 2013 and 2016, and other ion-ion or ion-proton collisions, at different energies.

In the coming years, an ambitious luminosity upgrade program, HL-LHC [30], with the accompanying LHC Injectors Upgrade [31], has as its target an order-of-magnitude increase in integrated luminosity through doubling the proton beam current, the utilization of new larger aperture $\mathrm{Nb_3Sn}$ superconducting final quadrupoles to allow squeezing the β^* to as low as 15 cm, superconducting compact *crab cavities* and luminosity leveling also for ATLAS and CMS as its key ingredients.

31.4.2 *Electron-Positron Rings*

Asymmetric energies of the two colliding beams cause the fast-moving reaction products to travel measurable distances before decaying, enabling more precise studies in, for example, B-meson physics. High-luminosity colliders of this type (B-factories, such as the former PEP-II at SLAC and KEKB in Japan) require advanced interaction-region designs and face numerous additional challenges [32]. SuperKEKB operates with 7 GeV electron and 4 GeV positron beams since 2018 and is aiming for luminosities of $(6-8)\times 10^{35}$ $\mathrm{cm^{-2}s^{-1}}$ [33]. In 2024, a world record luminosity of 5.1×10^{34} $\mathrm{cm^{-2}s^{-1}}$ has been reached, still at rather low beam currents. Vertical beam-beam tune shifts of $\xi_y \approx 0.05$ for the 4 GeV positron beam, and $\xi_y \approx 0.03$ for the 7 GeV electron beam have been achieved. These values are still about a factor of two lower than at the previous KEKB. Since 2020 SuperKEKB operates with a virtual *crab-waist* collision scheme, first developed for the FCC-ee design [34]. The original crab-waist scheme, based on additional sextupole magnets, was earlier implemented at DAΦNE [35]. The general crab-waist concept combines a large Piwinski angle Φ, and an extremely low β_y^* ($\ll \sigma_z$) with the cancellation of the transverse betatron resonances which occur under conditions of $kQ_x + lQ_y = n$, where k, l, n are integers. The latter is achieved by means of existing or additional electromagnetic sextupoles with special betatron phase advances to the collision point [36]. The crab-waist collision scheme has become a design choice for all proposed future e^+e^- circular colliders.

Beside SuperKEKB and DAΦNE, three other e^+e^- ring colliders currently in operation are VEPP-2000 with $\sqrt{s}$ up to 2.0 GeV, BEPC-II with $\sqrt{s}$ up to 4.6 GeV and VEPP-4M with maximum c.m.e. of 12 GeV [1].

31.4.3 *RHIC*

The Relativistic Heavy Ion Collider [37] employs 3.45 T Nb-Ti superconducting magnets, and collides combinations of fully-stripped ions such as H-H ($p-p$), p-Al, p-Au, d-Au, h-Au, Cu-Cu, Cu-Au, Zr-Zr, Ru-Ru, Au-Au, and U-U over a wide energy range. The high charge per particle (+79 for gold, for instance) makes intra-beam scattering of particles within the bunch a special concern, even for seemingly moderate bunch intensities. In 2012, 3-D stochastic cooling was successfully implemented in RHIC [38] and is now routinely used. With stochastic cooling, steady increases in the bunch intensity, and numerous other upgrades, RHIC now operates at 44 times the Au-Au design average luminosity. Unique among high energy colliders, RHIC heavy ions beams cross the *transition energy* $\gamma_{rmt} = 1/\sqrt{\alpha_\mathrm{c}}$ during acceleration – see Eqs.(31.11, 31.12) – a point where the derivative with respect to momentum of the revolution period is zero. This period of time is kept as short as allowed by the magnet ramp rate and must be dealt with carefully.

RHIC is also unique in its ability to accelerate and collide polarized proton beams. As proton beam polarization must be maintained from its low-energy source, successful acceleration through the myriad of depolarizing resonance conditions in high energy circular accelerators has taken years to accomplish [39]. An energy of 255 GeV per proton with 60% final polarization per beam has been realized. As part of a scheme to compensate the head-on beam-beam effect, electron lenses operated routinely during the record high beam-beam parameter polarized proton operation at 100 GeV energy in 2015 [40].

31.5 Future High-Energy Colliders and Prospects

Modern nuclear physics and high energy particle physics face critical questions which require next-generation high-energy and high-intensity experiments using hadron-hadron, lepton-lepton, and lepton-proton colliding-beam facilities. Understanding the structure of the proton and neutron directly from the dynamics of their quarks and gluons governed by the quantum chromodynamics calls for new ion-ion and electron-ion colliders. Two types of colliders are generally aspired by the HEP community [41, 42]: i) Higgs factories with a c.m.e. of 240–250 GeV in e^+e^- collisions for precision studies of the Higgs boson ($m_H = 125$ GeV) and exploration of the Higgs sector in greater detail, including measurements of Higgs couplings to fermions and vector bosons, self-coupling, rare decays, mass and width, that can also deliver other electroweak precision physics, e.g., on the Z-pole (91 GeV c.m.e.), at the W-pair threshold (about 160 GeV), and when run as a top quark factory (365–380 GeV); and ii) colliders with c.m.e. levels significantly beyond those of the LHC to explore the energy frontier for potential discoveries through direct searches in pp, $\mu\mu$, and e^+e^- interactions. In addition, precision physics at future high-luminosity factories operating at the τ-charm energy also provides sensitivity to new physics at multi-TeV energies and beyond. A comprehensive review of the future colliders' projects, ideas, and R&D activities can be found in Ref. [1]; Ref. [43] presents comparative analysis of implementation challenges of numerous future HEP collider proposals. Below we only briefly summarize leading collider proposals for construction over the next several decades which rely mostly on currently available technologies, such as NC or SC RF and/or NC or SC magnets, some of them requiring reasonable scope and duration mission-oriented development programs, as well as advanced schemes based on plasma acceleration and other innovative ideas. Tentative parameters of some of the colliders discussed, or mentioned, in this section are summarized in Table 31.1 and Table 31.2.

31.5.1 *Ion-Ion and Electron-Ion Colliders*

NICA (Nuclotron-based Ion Collider fAcility) is a new accelerator complex under construction at the Joint Institute for Nuclear Research (JINR, Dubna, Russia) to study the properties of hot and dense baryonic matter, strong interactions among quarks and gluons, and spin physics [44]. NICA will provide a range of beam species, from protons and polarized deuterons to heavy bismuth ions. The 503 m circumference SC superferric-magnet collider is designed to deliver an average luminosity in heavy-ion interactions at $\sqrt{s_{NN}} = 4\text{–}11$ GeV/u of $1 \times 10^{27}\text{cm}^{-2}\text{s}^{-1}$ for a variety of nuclei up to $^{209}\text{Bi}^{83+}$, and for polarized proton and deuteron collisions at $\sqrt{s} = 12\text{–}27$ GeV with $\mathcal{L} = (1\text{–}10) \times 10^{31}$ $\text{cm}^{-2}\text{s}^{-1}$. Major accelerator challenges at NICA include the suppression of strong intrabeam scattering and space-charge effects, which will be mitigated through extensive use of electron and stochastic cooling systems.

Also under construction is the Electron-Ion Collider (EIC) for nuclear physics research at Brookhaven National Laboratory in the US. It will collide a 41–275 GeV proton beam of the reconfigured RHIC with a 5–18 GeV electron beam circulating in a new storage ring [45]. The EIC physics requirements [46] call for electron and nucleon beams with high polarization, $P_{e,n} \sim 70\%$ (since the precision of measurements of interest scales as $\mathcal{L}P_e^2P_n^2$), a spectrum of ion beam species from deuterons to the heaviest nuclei (U or Pb), variable c.m.e. values from $\sqrt{s} = 20$ GeV to 140 GeV, high luminosities up to 10^{33-34} $\text{cm}^{-2}\text{s}^{-1}$, and the potential for multiple interaction regions. Major accelerator design challenges in achieving the required energy, luminosity, and polarization include the development of SRF crab cavities and advanced superconducting magnets for interaction region focusing, as well as high-intensity polarized particle sources complemented by specialized magnets and operational techniques—such as swap-out injection—to preserve polarization throughout acceleration and collision. Strong cooling of the EIC hadron beams is expected to be required if luminosity levels two orders of magnitude beyond those of the HERA ep collider are to be reached.

To that end, ring-based or ERL-based electron and stochastic cooling systems are under consideration, including advanced concepts such as *coherent electron cooling*, and *optical stochastic cooling* (see review [47] and references therein).

Table 31.1: Tentative parameters of selected future e^+e^- high-energy colliders. Parameters associated with different beam energy scenarios are comma-separated; H and V indicate horizontal and vertical directions.

	FCC-ee	CEPC	ILC	CLIC
Species	e^+e^-	e^+e^-	e^+e^-	e^+e^-
Beam energy E_b (GeV)	46, 120, 183	45.5, 120, 180	125, 250	190, 1500
Circumference or length (km)	90.66	100	20.5, 31	11, 50
Interaction regions	4	2	1	2, 1
Est. integrated luminosity per experiment (ab^{-1}/year)	17, 0.9, 0.17	15, 0.65, 0.07	0.2, 0.3	0.27, 0.34
Peak lumi. $\mathcal{L}$/IP (10^{34}cm^{-2}s^{-1})	144, 7.5, 1.45	115, 5.0, 0.5	1.4, 1.8	2.25, 3.6
Rep.rate (Hz, f_rev for rings)	3307	3000	5	100, 50
Polarization (%)	≥10, 0, 0	5–10, 0, 0	80/30 (e^-/e^+)	80/0 (e^-/e^+)
Time between collisions (μs)	0.025, 0.3, 2.5	0.025, 0.68, 2.6	0.55	0.0005
Energy spread (rms, 10^{-3})	1.15, 1.76, 1.86	1.3, 1.7, 2.0	e^-: 1.9, 1.2 e^+: 1.5, 0.7	3.5
Bunch length σ_z (rms, mm)	15.2, 5.6, 2.3	8.7, 4.1, 2.9	0.3	0.07, 0.044
IP beam size σ^* (rms, μm)	H: 9, 13, 37 V: 0.04, 0.03, 0.04	H: 6, 14, 39 V: 0.04, 0.04, 0.11	H: 0.52, 0.47 V: 0.008, 0.006	H: 0.15, 0.04 V: 0.003, 0.001
Emittance, ε_n (rms, μm)	H: 63, 155, 589 V: 0.19, 0.24, 0.47	H: 24, 150, 493 V: 0.12, 0.3, 1.7	H: 5, 10 V: 0.035, 0.035	H: 0.95, 0.66 V: 0.03, 0.02
β^* at interaction point (cm)	H: 11, 24, 90 V: 0.07, 0.1, 0.14	H: 13, 30, 104 V: 0.09, 0.1, 0.27	H: 1.3, 1.1 V: 0.041, 0.048	H: 0.8, 0.69 V: 0.01, 0.0068
Full crossing angle θ_c (mrad)	30	33	14	18/27, 21
Crossing scheme	crab waist	crab waist	crab crossing	crab crossing
Piwinski angle $\Phi = \sigma_z\theta_\mathrm{c}/(2\sigma_x^*)$	26, 6.6, 0.9	24, 4.9, 1.2	0	0
Beam-beam param. ξ_y (10^{-3})	98, 130, 144	127, 110, 100	n/a	n/a
Disruption parameter D_y	1.3, 0.9, 2.0	0.6, 1.3, 0.8	35, 25	13, 8
Average Upsilon Υ (10^{-2})	0.01, 0.04, 0.06	0.02, 0.04, 0.05	3, 6	17, 500
RF frequency f_RF (MHz)	400, 400, 800	650	1300	11994
Particles per bunch N (10^{10})	22, 17, 16	14, 13, 20	2	0.52, 0.37
Bunches per beam n_b	11200, 300, 60	11934, 268, 35	1312 (pulse)	352, 312 (trains at 50 Hz)
Average beam current I_b (mA)	1292, 27, 5.1	804, 16.7, 3.3	0.021	0.014, 0.009
Injection energy (GeV)	on E_b (top-up)	on E_b (top-up)	5.0 (linac)	9.0 (linac)
RF gradient G (MV/m)	10.6, 10.6, 20.2	17.4, 24.9, 27.6	31.5	72, 100
Total SR power loss (MW)	100	60	n/a	n/a
Total beam power (MW)	n/a	n/a	5.3, 10.5	5.6, 28
Key technology	—	—	high grad. SC RF	two-beam accel.

31.5.2 *Higgs/Electroweak Factories*

Higgs factory proposals generally aim at improving the precision of coupling measurements of Higgs boson, top quark, W and Z by an order of magnitude or more compared with previous studies. Two proposals for ∼100 km circumference circular e^+e^- colliders have recently gained momentum: the Future Circular Collider (FCC-ee) at CERN [48] and the Circular Electron-Positron Collider (CEPC) in China [49]. Design philosophy of these machines assumes use of the maximum RF power available to compensate $O(100\ \mathrm{MW})$ synchrotron radiation losses $P_\mathrm{SR} = 2I \cdot \Delta E_\mathrm{SR}$ and operation

Table 31.2: Tentative parameters of selected future high-energy hadronic and muon colliders. Parameters associated with different particle species for NICA and EIC, and different beam-energy scenarios for a muon collider, are comma-separated. Quantities are, where appropriate, r.m.s.; unless noted otherwise, energies refer to beam energy; H and V indicate horizontal and vertical directions. Parameters of HL-LHC can be found in the High-Energy Collider Parameters review tables.

	NICA	EIC	FCC-hh	SPPC	$\mu\mu$ collider
Species	ion-ion, *pp*	*ep*, *e*-ion	*pp*	*pp*	$\mu^+\mu^-$
Beam energy E_b (TeV)	0.0045/u, 0.013	0.01(e), 0.275(p)	42.5	62.5	0.063, 5
Circumference C (km)	0.503	3.834	90.66	100	0.3, 10
Interaction regions	1+1	1(2)	4	2	1, 2
Est. integr. luminosity per exp. (ab^{-1}/year)	$10^{-8,-3}$ (ii, pp)	0.1	0.9	0.6	0.001, 2.0
Peak luminosity $\mathcal{L}$ (10^{34}cm^{-2}s^{-1})	$10^{-7,-2}$ (ii, pp)	1.05	30	4.3	0.008, 20
Rep. rate (Hz, f_rev for rings)	$5.9{\cdot}10^5$	$7.8{\cdot}10^4$	3307	3000	15, 5
Time between collisions (μs)	0.077	0.010	0.025	0.025	1, 33
Energy spread (rms, 10^{-3})	1.5	0.6(e), 1(p)	0.1	0.1	0.04, 1
Bunch length σ_z (rms, mm)	600	7(e), 70(p)	80	60	63, 1.5
IP beam size σ^* (H/V rms, μm)	660	95/8.5	3.8 (init.)	3.0 (init.)	75, 0.9
Emittance ε_n (H/V rms, mm mrad)	0.8(i), 4 (p)	20/1.3(e), 11.3/1(p)	2.2 (init.)	1.2 (init.)	200, 25
Beta function at IP β^* (H/V cm)	60	45/5.6(e), 80/7.2(p)	30	50	1.7, 0.15
Beam-beam param. ξ (10^{-3} H/V)	4(i), 15 (p)	72/100(e),12(p)	5–15	15	22, 78
RF frequency f_RF (MHz)	39	591	400	400/200	352/704/1300
Particles per bunch N (10^{10})	0.3(i), 50(p)	17.2(e), 6.9(p)	10	4	400, 180
Bunches per beam n_b	22	1160	9500	10082	1
Average beam current I_b (mA)	480(i), 1000(p)	2500(e),1000(p)	500	190	640, 9(peak)
Injection energy (GeV)	1-3.8(i), 2-10(p)	on E_b(e), 25(p)	$\geq$1300	3200	on E_b
Peak magnetic field B (T)	1.8	0.248(e), 3.80(p)	14	20	14
Polarization (%)	0(i), >50(p)	> 70(e), >70(p)	0	0	0
SR power loss/beam (MW)	10^{-6}	10(e), $< 10^{-6}$(p)	1.2	2.2	$< 10^{-5}$, 0.16
Key technology	electron and stoch. cooling	inj. cooling stoch. cooling	Nb_3Sn/HTS magnets	HTS magnets	μ prod./cool. HTS magnets

at the beam-beam limit ξ_y that yields peak luminosity:

$$\mathcal{L} = \frac{3}{16\pi r_0^2 (m_e c^2)} \frac{P_\mathrm{SR} \xi_y \rho}{\beta_y^* \gamma^3} , \tag{31.17}$$

that scales approximately as $1/E_\mathrm{b}^{3.5}$ for practical limits on P, ξ_y and β_y^*. At high target luminosities, the short beam lifetime caused by radiative Bhabha scattering necessitates the construction of a full-energy injector ring in the same tunnel, allowing continuous *top-up* injection of the electron and positron bunches in the collider rings operating at constant energy. Beamstrahlung imposes an additional limitation on beam lifetime, depending on momentum acceptance—making sufficient off-momentum dynamic aperture one of the design challenges—and also leads to some bunch lengthening.

These ambitious, large-scale projects based on well-established technologies are not extendable to TeV or multi-TeV energies, but they offer several important advantages that include the potential for much higher luminosities, and, thus, higher precision, the ability to operate multiple experiments simultaneously, and their ∼100 km circular tunnels that could later house $O(100$ TeV$)$ hadron colliders. The high energy efficiency of circular e^+e^- colliders is further boosted by advances in RF power sources, by improved SC cavities, and by innovative low-power magnet systems including ones based on high-temperature superconductors (HTS) at moderate magnetic field.

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For more than four decades, efforts have been devoted to developing high-gradient RF technology linear e^+e^- colliders in order to overcome the synchrotron radiation limitations of circular e^+e^- machines. The International Linear Collider (ILC), with a c.m.e. of 250 GeV in e^+e^- collisions, has been under consideration for more than two decades and could potentially be upgraded to $\sqrt{s}$=500 GeV and further to 1 TeV [50]. CERN's Compact Linear Collider (CLIC) design, developed since the mid-1980s, also includes possible upgrades, from an initial 380 GeV c.m.e. to ultimately 3 TeV, which would enable searches for new particles of significantly higher masses [51].

The primary challenge for a high-energy, high-luminosity single-pass collider design is the beam power requirement, so that measures must be taken to keep the power demand within bounds as illustrated in a transformed Eq.(31.15):

$$\mathcal{L} = \frac{1}{8\pi\alpha r_0}\,\frac{P_{\rm wall}}{\sqrt{s}}\,\frac{\eta}{\sigma_y^*}\,N_\gamma\,H_{\rm D}\,. \tag{31.18}$$

Here, $P_{\rm wall}$ is the total wall-plug power of the collider, to be converted into beam power $P_{\rm b} = 2f_0NE_{\rm b}$ with efficiency η, $N_\gamma \approx 2\alpha r_0N/\sigma_x^*$ is the number of beamstrahlung photons emitted per $e^\pm$ ($\alpha \approx 1/137$ denotes the fine-structure constant), and the last factor $H_{\rm D}$, typically between 1 and 2, represents the enhancement of luminosity due to the *pinch effect*, the additional focusing occurring during the collision of oppositely charged bunches. The management of $P_{\rm wall}$ leads to an upward push on the bunch population N with an attendant rise in the energy radiated due to the electromagnetic field of one bunch acting on the particles of the other (beamstrahlung). Keeping a significant fraction of the luminosity close to the nominal energy represents a design goal, which is met if N_γ does not exceed a value of about 1. A consequence is the use of flat beams, where N_γ is managed by the beam width, and luminosity adjusted by the beam height, thus the explicit appearance of the vertical beam size σ_y^*.

The ILC [52] is based on 1.3 GHz standing-wave superconducting accelerating structures operating at 2 K with 31.5 MV/m average gradient, up to 8 nm vertical beam size at the IP, and luminosity comparable to the LHC. Progress toward higher field gradients and Q values continues to be made, with nitrogen-doping techniques being a recent example [53]. Taking advantage of such progress, a Linear Collider Facility (LCF) at CERN is proposed, which, in a first superconducting stage, could deliver two times the ILC luminosity to each of two experiments [54]. CLIC is based on a novel two-beam acceleration scheme [55]. Here, room-temperature NC copper high-gradient 12 GHz accelerating structures are powered by a high-current 1.9 GeV drive beam to efficiently enable accelerating gradients of up to 100 MV/m (though optimal G=70 MV/m for the first CLIC stage at $\sqrt{s}$=380 GeV, and for this stage an alternative RF power drive option with 12 GHz klystrons powering is also being considered).

To reach their design luminosities, linear colliders require unprecedented rates of positron production, about (20–100) times the world record set by the SLC positron source, mitigation of wakefield effects, advanced beam-based trajectory tuning, and very tight control of imperfections, misalignments, and jitter [56] (for example, for CLIC, this entails $O(10\ \mu\text{m})$ accuracy of pre-alignment of the main linac and beam delivery system components, and suppression of fast quadrupole vibrations from ground motion to the $O(1\ \text{nm})$ level at frequencies above 1 Hz; the ILC tolerances are more relaxed, so that a linac pre-alignment of 200–300 μm suffices, and vibrations must be suppressed to the $O(1\ \text{nm})$ level only above 1 MHz).

Recent developments in RF technologies offer the promise of more compact linear e^+e^- Higgs/-Electroweak factories, based either on ∼70 MV/m traveling-wave 1.3 GHz SRF structures [57] or on 70–120 MV/m high-efficiency, distributed-RF-coupling, normal-conducting cold-copper 5.7 GHz accelerating cavities operated at liquid nitrogen temperature [58].

There are a number of alternative ideas proposed for studies of the Higgs/Electroweak physics, such as high-energy, high-luminosity e^+e^- collider in a 100 km tunnel using ERLs to accelerate particles to collision energy in 4 to 6 passes and return up to 81% of the energy back into the SRF cavities on deceleration turns, thus, lowering the required facility power several-fold; similar power recovery in one pass can greatly improve efficiency of linear colliders; an arrangement of $\gamma\gamma$ collision through near-IP conversion of high energy electron beams into intense photon beams by backward Compton scattering off a high-power laser or off an FEL photon pulse; $\mu\mu$ Higgs factory with unprecedented 0.004% energy resolution, and a high-energy lepton-hadron collider bringing into collision a 50- or 60-GeV electron beam from an ERL with the 7 TeV protons circulating in the LHC (LHeC) (see [1] and references therein). At lower energies, Super Tau-Charm Factory proposals aim at the production and precise study of charmonium states and of the tau lepton [59, 60].

31.5.3 *Energy Frontier Circular Colliders*

Several hadron and lepton colliders have been proposed to extend the energy reach beyond the LHC. As noted above, ambitious plans have been proposed to upgrade the FCC and CEPC to hadron colliders – FCC-hh at CERN and Super Proton Proton Collider (SPPC) in China, respectively – by means of next- or next-next generation SC magnets installed in the arc sections of the 100 km rings, so as to enable $\sqrt{s}$ of the order 100 TeV or above [48, 49, 61]. Comparable discovery reach is expected for a circular 10–14 TeV muon collider [62], significantly beyond that of practical e^+e^- linear colliders.

The maximum beam energy Eq.(31.1) is directly proportional to the magnetic field and to the ring circumference, hence, future hadron colliders like FCC-hh and SPPC rely on the development of the technology of 14–16 T Nb_3Sn dipole magnets [63] or dipole magnets based on high-temperature superconductors (HTS) with a field of up to 18-20 T, including iron-based HTS [64]. Though higher fields are possible with HTS, more cost-effective might be hybrid magnet designs incorporating Nb-Ti, Nb_3Sn, and an inner layer of HTS and providing fields of about 20 T. Another important technology is the cryogenic beam vacuum system, which has to cope with unusually high levels of synchrotron radiation (up to 15 MW in total, for FCC-hh) in a cold environment. The beam-screen intercepting the radiation inside the cold bore of the magnets should operate at or above 50 K — significantly higher temperature than in the LHC.

Design luminosities of these hadron colliders $O(10^{35}\ \text{cm}^{-2}\text{s}^{-1})$ will result in a pile-up of events per crossing $O(500)$ (from up to 60 in LHC) and fast intensity drop due to burn-off. Significant radiation damping of beam emittances will naturally level luminosity evolution, though the total beam-beam tune shift $n_{\text{IP}}\xi \sim (0.01 - 0.03)$ might need a special control as it will increase during the store [65].

Future hadron colliders store enormous electromagnetic energy in their superconducting dipole magnets, posing a major challenge for quench protection. Moreover, the record-high stored beam energy — about ∼13 GJ in the FCC-hh — makes machine protection systems an equally critical concern. A very challenging multi-stage collimation system is needed to avoid local beam loss spikes near cold magnets, which would induce magnet quenches. The primary and secondary collimators of the LHC are based on carbon-carbon composite material. For the future hadron colliders, ever stronger materials are being developed and examined, which also feature higher conductivity and, hence, lower impedance. More advanced options include the use of short bent crystals as primary collimators [66] and the deployment of hollow electron-beam lenses as non-destructible collimators [67]. It is noteworthy that machines such as the FCC-hh or SPPC can also be employed for particle physics studies in ion-ion and ion-proton collisions as well as in lepton-hadron ($ep, \mu p$) interactions with c.m.e. O(3–10 TeV), should the corresponding high-energy high-intensity electron or muon beam facilities (ring-based, ERL-based or linacs) be found feasible and practical.

The lifetime of the muon, $\gamma\tau_0$ (with $\tau_0 = 2.2\ \mu$s), is sufficient to allow rapid acceleration to high energies before the muons decay, and to permit storage in a ring for approximately $300 \times B$[T] turns, where B is the average dipole magnetic field in Tesla. The muon-to-electron mass ratio of 210 effectively eliminates the synchrotron radiation barrier, allowing a muon collider to be built at a scale compatible with existing accelerator laboratory sites. High-energy muon colliders, as presently conceived, are predicted to be more compact, more power-efficient and significantly less expensive than the equivalent energy-frontier hadron or e^+e^- machines [62], and a neutrino factory could potentially be realized in the course of their construction [68]. The Higgs production cross-section in the s-channel is enhanced by a factor of $(m_\mu/m_e)^2$ compared to that in e^+e^- collisions.

The average luminosity of a muon collider,

$$\langle \mathcal{L} \rangle = f_0 \gamma^2 \frac{c\tau_0}{2C} \frac{n_\mathrm{b} N^2}{4\pi\varepsilon_\mathrm{n}\beta^*}\mathcal{F} = B P_\mathrm{b} \frac{N r_0}{4\pi\varepsilon_\mathrm{n}} \frac{\gamma}{\beta^*}\left(\frac{c\tau_0\mathcal{F}}{8\pi e}\right), \tag{31.19}$$

scales with B, the total beam power P_b, and the beam brightness – the third factor above is nothing but the muon beam-beam tune shift Eq. (31.16). There is an obvious incentive to have all the particles in just one bunch per beam. The beta-function at the two IPs $\beta^* \approx \sigma_z$ scales as $1/\gamma$ within certain range of energies, giving overall scaling $\langle \mathcal{L} \rangle \propto \gamma^2$ with other limiting parameters fixed. The main challenges to luminosity achievement with decaying particles are related to production and fast cooling and acceleration of $O(10^{12})$ muons per bunch without emittance degradation. A multi-TeV c.m.e. high luminosity $O(10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1})$ muon collider would consist of [69] : (i) a high power proton driver (e.g., 8 GeV 2-4 MW H$^-$ SRF linac); (ii) pre-target accumulation and compressor rings, in which high intensity 1-3 ns long proton bunches are formed; (iii) a high-power target for converting the proton beam into a tertiary muon beam with energy of about 200 MeV; (iv) a multi-stage ionization cooling section that reduces transverse and longitudinal emittances by several orders of magnitude and creates a low emittance beam, similarly to that recently demonstrated [70]; (v) a multistage acceleration system, possibly employing either rapid cycling synchrotrons or RLAs to accelerate muons in a modest number of turns up to the final energy using superconducting RF technology; and, finally, (vi) a 3–14 km diameter collider ring, where counter-propagating muon beams are stored and collide over the roughly 3000 turns corresponding to the muon lifetime.

The intense neutrino flux originating from the multi-TeV muon beams decaying in the collider poses another challenge — the need to minimize the environmental impact. The collider complex is typically located underground, and when the produced neutrinos emerge at the surface, a small fraction interacts with the surrounding rock (and other materials), generating an ionizing radiation dose that scales approximately as γ^3 [71]. Proposed facility designs aim to fully mitigate the impact of this neutrino-induced radiation to remain within operationally permitted levels. This can be achieved through a combination of measures, such as continuously adjusting the beam orbits to spread the neutrino flux over a wider area, constructing deeper collider tunnels, or reducing the muon beam emittance so that the desired luminosity can be reached with a substantially smaller number of muons.

31.5.4 *Plasma Acceleration and Other Advanced Concepts*

Since about the mid-1950s, it has been understood that collective plasma-based accelerators promise extremely large accelerating gradients, approximately three orders of magnitude greater than ∼100 MV/m obtained in conventional breakdown-limited RF structures [72]. Ionized plasmas can sustain electron plasma density waves with electric fields in excess of $E_0 = cm_e\omega_\mathrm{p}/e$ or

$$E_0 \simeq 96\,[\mathrm{GV/m}]\,\sqrt{n_0[10^{18}\,\mathrm{cm}^{-3}]}, \tag{31.20}$$

where n_0 denotes the ambient electron number density and $\omega_p = \sqrt{e^2 n_0/(m_e \varepsilon_0)}$ is the electron plasma frequency [73]. Such gradients can be effectively excited by either powerful external pulses of laser light or by electron bunches if they are shorter than the plasma wavelength $\lambda_p = c/\omega_p \simeq 33\ \mu\mathrm{m}\times\sqrt{10^{18}\,\mathrm{cm}^{-3}/n_0}$, or by longer beams of protons if their charge density is modulated with the period of λ_p.

In the past decade, impressive progress has been made in the plasma wakefield acceleration (PWA) of good-quality electron beams. Laser-driven electron energy gains of 10 GeV over 0.1-0.2 m of plasma with (1-6)$\times10^{17}\mathrm{cm}^{-3}$ density has been demonstrated by the UTA, UMD and CSU groups [74, 75], and at the BELLA facility at the Lawrence Berkeley National Laboratory (LBNL) [76]. Short electron bunches were used to boost the energy of externally injected electron bunches by 9 GeV over 1.3 m of $\sim10^{17}\mathrm{cm}^{-3}$ plasma at the FACET facility in SLAC [77]. The AWAKE experiment at CERN used long self-modulated 450 GeV proton bunches to accelerate electrons to 2 GeV over 10 m of $\sim10^{15}\ \mathrm{cm}^{-3}$ plasma [78].

Despite recent advances, the applicability of plasma acceleration in a high-energy physics facility remains uncertain. The design of the most-sought multi-TeV PWA-based e^+e^- collider faces numerous challenges. The most fundamental issue is the acceleration of positively charged positrons, which, unlike electrons, are strongly defocused by the immobile ions in the electron-depleted plasma-wakefield channel. While several strategies to mitigate this defocusing are being explored, a few alternative plasma-based collider concepts have been proposed to circumvent the positron problem entirely, including asymmetric plasma–RF hybrid colliders, e^-e^- and $\gamma\gamma$ colliders (see [79] and references therein). Furthermore, the ultra-strong focusing gradients in the plasma—focusing for e^- and defocusing for e^+—make the transfer of highly divergent beams between PWA stages extremely challenging, requiring significant space for beam optics, and effectively reducing the net accelerating gradient [80]. The luminosity of PWA-based colliders, still described by Eq. (31.18), fundamentally hinges on high power efficiency η, and also requires excellent beam quality, tight control of coherent beam instabilities, and stringent alignment and vibration tolerances—yet by any metrics, it remains orders of magnitude below that projected for colliders based on conventional technology (e.g., CLIC) [79, 81].

In addition, many advanced approaches and innovative concepts have been proposed to extend the energy and physics reach of future particle colliders, reduce their cost, and enhance luminosity or energy efficiency. Among these are: i) *dielectric wakefield accelerators*, where resonant dielectric structures are powered by ultra-short RF wakefield pulses driven by collinear or preceding high-charge electron bunches, sustaining gradients up to ∼0.3 GV/m; ii) *dielectric laser accelerators*, micron-scale dielectric structures driven by laser pulses, supporting fields of order $O(1\ \mathrm{GV/m})$; iii) *linear muon crystal colliders*, exploiting $O(10\ \mathrm{TV/m})$ wakefield acceleration of muons channeling between crystal planes or inside carbon nanotubes with charge carrier densities $\sim10^{20-22}\ \mathrm{cm}^{-3}$, potentially enabling ultimate compact PeV-scale facilities; iv) the *Gamma Factory* concept, in which conventional laser pulses collide with partially stripped heavy-ion beams circulating in a hadron storage ring (e.g., the LHC or FCC-hh), producing intense bursts of double–Lorentz-boosted high-energy gamma rays that could enable the generation of positrons or muons at unprecedented rates.

An overview of these and related concepts, along with extensive references, can be found, e.g., in Ch. V of [1].

The feasibility of applying PWA and other advanced technologies to future high-energy colliders is being explored through worldwide R&D programs.

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