

74.  $B^0-\bar{B}^0$  Mixing

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There are two neutral  $B^0-\bar{B}^0$  meson systems,  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  (generically denoted  $B_q^0-\bar{B}_q^0$ ,  $q = s, d$ ), which exhibit particle-antiparticle mixing [1]. This mixing phenomenon is described in Ref. [2]. In the following, we adopt the notation introduced in Ref. [2], and assume  $CPT$  conservation throughout. In each system, the light (L) and heavy (H) mass eigenstates,

$$|B_{qL,qH}\rangle = p|B_q^0\rangle \pm q|\bar{B}_q^0\rangle, \quad (74.1)$$

have a mass difference  $\Delta m_q = m_{qH} - m_{qL} > 0$ , a total decay-width difference  $\Delta\Gamma_q = \Gamma_{qL} - \Gamma_{qH}$  and an average decay width  $\Gamma_q = (\Gamma_{qL} + \Gamma_{qH})/2$ . In the absence of  $CP$  violation in the mixing,  $|q/p| = 1$ , the differences are given by  $\Delta m_q = 2|M_{12}|$  and  $|\Delta\Gamma_q| = 2|\Gamma_{12}|$ , where  $M_{12}$  and  $\Gamma_{12}$  are the off-diagonal elements of the mass and decay matrices [2]. The evolution of a pure  $|B_q^0\rangle$  or  $|\bar{B}_q^0\rangle$  state at  $t = 0$  is given by

$$|B_q^0(t)\rangle = g_+(t)|B_q^0\rangle + \frac{q}{p}g_-(t)|\bar{B}_q^0\rangle, \quad (74.2)$$

$$|\bar{B}_q^0(t)\rangle = g_+(t)|\bar{B}_q^0\rangle + \frac{p}{q}g_-(t)|B_q^0\rangle, \quad (74.3)$$

which means that the flavor states remain unchanged (+) or oscillate into each other (−) with time-dependent probabilities proportional to

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma_q t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma_q}{2}t\right) \pm \cos(\Delta m_q t) \right]. \quad (74.4)$$

In the absence of  $CP$  violation, the time-integrated mixing probability  $\int |g_-(t)|^2 dt / (\int |g_-(t)|^2 dt + \int |g_+(t)|^2 dt)$  is given by

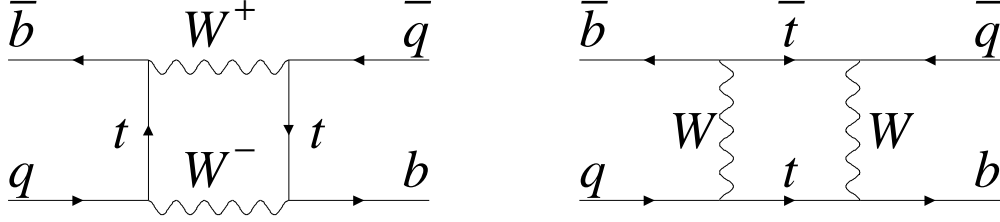
$$\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2 + 1)}, \quad \text{where} \quad x_q = \frac{\Delta m_q}{\Gamma_q}, \quad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}. \quad (74.5)$$

### 74.1 Standard Model predictions and phenomenology

In the Standard Model, the transitions  $B_q^0 \rightarrow \bar{B}_q^0$  and  $\bar{B}_q^0 \rightarrow B_q^0$  are due to the weak interaction. They are described, at the lowest order, by box diagrams involving two  $W$  bosons and two up-type quarks (see Fig. 74.1), as is the case for  $K^0 - \bar{K}^0$  mixing. However, the long range interactions arising from intermediate virtual states are negligible for the neutral  $B$  meson systems, because the large  $B$  mass is off the region of hadronic resonances. The calculation of the dispersive and absorptive parts of the box diagrams yields the following predictions for the off-diagonal element of the mass and decay matrices [3],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} S_0(m_t^2/m_W^2) (V_{tq}^* V_{tb})^2, \quad (74.6)$$

$$\begin{aligned} \Gamma_{12} = & \frac{G_F^2 m_b^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi} \\ & \times \left[ (V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb} \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) \right. \\ & \left. + (V_{cq}^* V_{cb})^2 \mathcal{O}\left(\frac{m_c^4}{m_b^4}\right) \right], \quad (74.7) \end{aligned}$$



**Figure 74.1:** Dominant box diagrams for the  $B_q^0 \rightarrow \bar{B}_q^0$  transitions ( $q = d$  or  $s$ ). Similar diagrams exist where one or both  $t$  quarks are replaced with  $c$  or  $u$  quarks.

where  $G_F$  is the Fermi constant,  $m_W$  the  $W$  boson mass, and  $m_i$  the mass of quark  $i$ ;  $m_{B_q}$ ,  $f_{B_q}$  and  $B_{B_q}$  are the  $B_q^0$  mass, weak decay constant and bag parameter, respectively. The known function  $S_0(x_t)$  can be approximated very well by  $0.784 x_t^{0.76}$  [4], and  $V_{ij}$  are the elements of the CKM matrix [5]. The QCD corrections  $\eta_B$  and  $\eta'_B$  are of order unity. The only non-negligible contributions to  $M_{12}$  are from box diagrams involving two top quarks. The phases of  $M_{12}$  and  $\Gamma_{12}$  satisfy

$$\phi_M - \phi_\Gamma = \pi + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right), \quad (74.8)$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the  $K^0-\bar{K}^0$  system, the heavy state is expected to have a smaller decay width than that of the light state:  $\Gamma_H < \Gamma_L$ . Hence,  $\Delta\Gamma_q = \Gamma_L - \Gamma_H$  is expected to be positive in the Standard Model.

Furthermore, the quantity

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2} \frac{m_b^2}{m_W^2} \frac{1}{S_0(m_t^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right) \quad (74.9)$$

is small, and a power expansion of  $|q/p|^2$  yields

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\phi_M - \phi_\Gamma) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right). \quad (74.10)$$

Therefore, considering both Eqs. (74.8) and (74.9), the  $CP$ -violating parameter

$$1 - \left|\frac{q}{p}\right|^2 \simeq \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \quad (74.11)$$

is expected to be very small:  $\sim \mathcal{O}(10^{-3})$  for the  $B_d^0-\bar{B}_d^0$  system and  $\lesssim \mathcal{O}(10^{-4})$  for the  $B_s^0-\bar{B}_s^0$  system [6].

In the approximation of negligible  $CP$  violation in mixing, the ratio  $\Delta\Gamma_q/\Delta m_q$  is equal to the small quantity  $|\Gamma_{12}/M_{12}|$  of Eq. (74.9); it is hence independent of CKM matrix elements, *i.e.*, the same for the  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  systems. Calculations [7] yield  $\sim 5 \times 10^{-3}$  with a  $\sim 15\%$  uncertainty. Given the published experimental knowledge [8] on the mixing parameter  $x_q$

$$\begin{cases} x_d = 0.7659 \pm 0.0032 & (B_d^0-\bar{B}_d^0 \text{ system}) \\ x_s = 26.91 \pm 0.11 & (B_s^0-\bar{B}_s^0 \text{ system}) \end{cases}, \quad (74.12)$$

the Standard Model thus predicts that  $\Delta\Gamma_d/\Gamma_d$  is very small (below 1%), but  $\Delta\Gamma_s/\Gamma_s$  considerably larger ( $\sim 10\%$ ). These width differences are caused by the existence of final states to which both the  $B_q^0$  and  $\bar{B}_q^0$  mesons can decay. Such decays involve  $b \rightarrow c\bar{c}q$  quark-level transitions, which are Cabibbo-suppressed if  $q = d$  and Cabibbo-allowed if  $q = s$ .

Complete sets of Standard Model predictions for all mixing parameters in both the  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  systems can be found in Refs. [7, 9].

## 74.2 Experimental issues and methods for oscillation analyses

Time-integrated measurements of  $B^0-\bar{B}^0$  mixing were published for the first time in 1987 by UA1 [10] and ARGUS [11], and since then by other experiments. These measurements are typically based on counting same-sign and opposite-sign lepton pairs from the semileptonic decay of the produced  $b\bar{b}$  pairs. Such analyses cannot easily separate the contributions from the different  $b$ -hadron species, therefore, the clean environment of  $\Upsilon(4S)$  machines (where only  $B_d^0$  and charged  $B_u$  mesons are produced) is in principle best suited to measure  $\chi_d$ .

However, better sensitivity is obtained from time-dependent analyses aiming at the direct measurement of the oscillation frequencies  $\Delta m_d$  and  $\Delta m_s$ , from the proper time distributions of  $B_d^0$  or  $B_s^0$  candidates identified through their decay in (mostly) flavor-specific modes, and suitably tagged as mixed or unmixed. This is particularly true for the  $B_s^0-\bar{B}_s^0$  system, where the large value of  $x_s$  implies maximal mixing, *i.e.*,  $\chi_s \simeq 1/2$ . In such analyses, the  $B_d^0$  or  $B_s^0$  mesons are either fully reconstructed, partially reconstructed from a charm meson, selected from a lepton with the characteristics of a  $b \rightarrow \ell^-$  decay, or selected from a reconstructed displaced vertex. At high-energy colliders (LEP, SLC, Tevatron, LHC), the proper time  $t = \frac{m_B}{p}L$  is measured from the distance  $L$  between the production vertex and the  $B$  decay vertex, and from an estimate of the  $B$  momentum  $p$ . At asymmetric  $B$  factories (PEP-II, KEKB, SuperKEKB), producing  $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d^0 \bar{B}_d^0$  events with a boost  $\beta\gamma$  ( $= 0.55, 0.425, 0.28$ ), the proper time difference between the two  $B$  candidates is estimated as  $\Delta t \simeq \frac{\Delta z}{\beta\gamma c}$ , where  $\Delta z$  is the spatial separation between the two  $B$  decay vertices along the boost direction. In all cases, the good resolution needed on the vertex positions is obtained with silicon detectors.

The average statistical significance  $\mathcal{S}$  of a  $B_q^0$  oscillation signal can be approximated as [12]

$$\mathcal{S} \approx \sqrt{N/2} f_{\text{sig}} (1 - 2\eta) e^{-(\Delta m_q \sigma_t)^2/2}, \quad (74.13)$$

where  $N$  is the number of selected and tagged candidates,  $f_{\text{sig}}$  is the fraction of signal in that sample,  $\eta$  is the total mistag probability, and  $\sigma_t$  is the resolution on proper time (or proper time difference). The quantity  $\mathcal{S}$  decreases very quickly as  $\Delta m_q$  increases; this dependence is controlled by  $\sigma_t$ , which is therefore a critical parameter for  $\Delta m_s$  analyses. At high-energy colliders, the proper time resolution  $\sigma_t \sim \frac{m_B}{\langle p \rangle} \sigma_L \oplus t \frac{\sigma_p}{p}$  includes a constant contribution due to the decay length resolution  $\sigma_L$  (typically 0.04–0.3 ps), and a term due to the relative momentum resolution  $\sigma_p/p$  (typically 10–20% for partially reconstructed decays), which increases with proper time. At  $B$  factories, the boost of the  $B$  mesons is estimated from the known beam energies, and the term due to the spatial resolution dominates (typically 0.7–1.5 ps because of the much smaller  $B$  boost).

In order to tag a  $B_q^0$  candidate as mixed or unmixed, it is necessary to determine its flavor both in the initial state and in the final state. The initial and final state mistag probabilities,  $\eta_i$  and  $\eta_f$ , degrade  $\mathcal{S}$  by a total factor  $(1 - 2\eta) = (1 - 2\eta_i)(1 - 2\eta_f)$ . In lepton-based analyses, the final state is tagged by the charge of the lepton from  $b \rightarrow \ell^-$  decays; the largest contribution to  $\eta_f$  is then due to  $\bar{b} \rightarrow \bar{c} \rightarrow \ell^-$  decays. Alternatively, the charge of a reconstructed charm meson ( $D^{*-}$  from  $B_d^0$  or  $D_s^-$  from  $B_s^0$ ), or that of a kaon hypothesized to come from a  $b \rightarrow c \rightarrow s$  decay [13], can be used. For fully-inclusive analyses based on topological vertexing, final-state tagging techniques include

jet-charge [14] and charge-dipole [15,16] methods. At high-energy colliders, the methods to tag the initial state (*i.e.*, the state at production), can be divided into two groups: the ones that tag the initial charge of the  $\bar{b}$  quark contained in the  $B_q^0$  candidate itself (same-side tag), and the ones that tag the initial charge of the other  $b$  quark produced in the event (opposite-side tag). On the same side, the sign of a charged pion, kaon or proton from the primary vertex is correlated with the production state of the  $B_q^0$  meson if that particle is a decay product of a  $B^{**}$  state or the first in the fragmentation chain [17,18]. Jet- and vertex-charge techniques work on both sides and on the opposite side, respectively. Finally, the charge of a lepton from  $b \rightarrow \ell^-$ , of a kaon from  $b \rightarrow c \rightarrow s$  or of a charm hadron from  $b \rightarrow c$  [19] can be used as an opposite-side tag, keeping in mind that its performance is degraded due to integrated mixing. At SLC, the beam polarization produced a sizeable forward-backward asymmetry in the  $Z \rightarrow b\bar{b}$  decays, and provided another very interesting and effective initial state tag based on the polar angle of the  $B_q^0$  candidate [15]. Initial state tags have also been combined to reach  $\eta_i \sim 26\%$  at LEP [18,20] or 22% at SLD [15] with full efficiency. In the case  $\eta_f = 0$ , this corresponds to an effective tagging efficiency  $Q = \epsilon D^2 = \epsilon(1 - 2\eta)^2$ , where  $\epsilon$  is the tagging efficiency, in the range 23 – 31%. The equivalent figure achieved by CDF during Tevatron Run I was  $\sim 3.5\%$  (see tagging summary on page 160 of Ref. [21]), reflecting the fact that tagging is more difficult at hadron colliders. The CDF and DØ analyses of Tevatron Run II data reached  $\epsilon D^2 = (1.8 \pm 0.1)\%$  [22] and  $(2.5 \pm 0.2)\%$  [23] for opposite-side tagging, while same-side kaon tagging (for  $B_s^0$  analyses) contributed an additional 3.7 – 4.8% at CDF [22], and pushed the combined performance to  $(4.7 \pm 0.5)\%$  at DØ [24]. LHCb, operating in the forward region at the LHC where the environment is different in terms of track multiplicity and  $b$ -hadron production kinematics, has reported  $\epsilon D^2 = (2.10 \pm 0.25)\%$  [25] for opposite-side tagging,  $(1.80 \pm 0.26)\%$  [26] for same-side kaon tagging, and  $(2.11 \pm 0.11)\%$  [27] for same-side pion and proton tagging: the combined figure ranges typically between  $(3.73 \pm 0.15)\%$  [28] and  $(6.3 \pm 0.5)\%$  [29] depending on the mode in which the tagged  $B_s^0$  meson is reconstructed, and reaches up to  $(8.1 \pm 0.6)\%$  [30] for hadronic  $B_d^0$  modes. ATLAS [31] and CMS [32] have reported  $\epsilon D^2 \sim 1.75\%$  and  $\epsilon D^2 \sim 5.6\%$ , respectively, using opposite-side tagging of  $B_s^0 \rightarrow J/\psi\phi$  decays.

At  $B$  factories, the flavor of a  $B_d^0$  meson at production cannot be determined, since the two neutral  $B$  mesons produced in a  $\Upsilon(4S)$  decay evolve in a coherent  $P$ -wave state where they keep opposite flavors at any time. However, as soon as one of them decays, the other follows a time-evolution given by Eqs. (74.2) or (74.3), where  $t$  is replaced with  $\Delta t$  (which will take negative values half of the time). Hence, the “initial state” tag of a  $B$  can be taken as the final-state tag of the other  $B$ . Effective tagging efficiencies of 30% are achieved by BaBar and Belle [33], using different techniques including  $b \rightarrow \ell^-$  and  $b \rightarrow c \rightarrow s$  tags. It is worth noting that, in this case, mixing of the other  $B$  (*i.e.*, the coherent mixing occurring before the first  $B$  decay) does not contribute to the mistag probability.

Before the experimental observation of a decay-width difference, oscillation analyses typically neglected  $\Delta\Gamma_q$  in Eq. (74.4), and described the time dependence with the functions  $\Gamma_q e^{-\Gamma_q t} (1 \pm \cos(\Delta m_q t))/2$  (high-energy colliders) or  $\Gamma_d e^{-\Gamma_d |\Delta t|} (1 \pm \cos(\Delta m_d \Delta t))/4$  (asymmetric  $\Upsilon(4S)$  machines). As can be seen from Eq. (74.4), a non-zero value of  $\Delta\Gamma_q$  would effectively reduce the oscillation amplitude with a small time-dependent factor that would be very difficult to distinguish from time resolution effects. Measurements of  $\Delta m_q$  are usually extracted from the data using a maximum likelihood fit.

### 74.3 $\Delta m_d$ and $\Delta\Gamma_d$ measurements

Many  $B_d^0-\bar{B}_d^0$  oscillation analyses have been published [34] by the ALEPH [35], DELPHI [16,36], L3 [37], OPAL [38,39], CDF [17], DØ [23], BaBar [40], Belle [41], Belle II [42], and LHCb [43–46] collaborations. Although a variety of different techniques have been used, the individual

$\Delta m_d$  results obtained at LEP and Tevatron have remarkably similar precision. Their average is compatible with the more precise measurements at the asymmetric  $B$  factories and the LHC. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or  $b$ -hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the  $b$ -hadron lifetimes and fractions published in this *Review*. Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fragmentation fractions, lifetimes, branching ratios of  $b$  hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, tagging, background description). Combining all measurements [16, 17, 23, 35–46] and accounting for all identified correlations yields  $\Delta m_d = 0.5068 \pm 0.0016(\text{stat}) \pm 0.0011(\text{syst}) \text{ ps}^{-1}$  [8], a result dominated by the latest LHCb measurement with  $B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu X$  decays [46].

The decay-width difference can be determined from the  $B^0$  effective lifetimes [47, 48]

$$\tau_d^{\text{eff}} = \frac{1 + 2\mathcal{A}_{\Delta\Gamma} y_d + y_d^2}{\Gamma_d(1 - y_d^2)(1 + \mathcal{A}_{\Delta\Gamma} y_d)}, \quad (74.14)$$

where  $\mathcal{A}_{\Delta\Gamma} = 0$  for a flavor-specific final state and  $\mathcal{A}_{\Delta\Gamma} \simeq \eta_{CP} \cos(2\beta)$  for a  $CP$  eigenstate with eigenvalue  $\eta_{CP}$ . A fit to all available effective  $B^0$  lifetime measurements using flavor-specific final states or the  $CP$ -odd  $J/\psi K_S^0$  final state, as well as the dedicated measurements of  $\Delta\Gamma_d/\Gamma_d$  by BaBar [49], Belle [50], LHCb [51], ATLAS [52] and CMS [53], using the external constraint  $\beta \equiv \arg(-V_{cd}V_{cb}^*/(V_{td}V_{tb}^*)) = 22.63 \pm 0.45$  degrees [8], allows the simultaneous determination [8] of

$$\Delta\Gamma_d/\Gamma_d = +0.005 \pm 0.010 \quad \text{and} \quad 1/\Gamma_d = 1.5113 \pm 0.0027 \text{ ps}. \quad (74.15)$$

These results are compatible both with  $\Delta\Gamma_d = 0$  and with the Standard Model prediction for  $\Delta\Gamma_d$  of  $0.00215 \pm 0.00013 \text{ ps}^{-1}$  [9].

The time-integrated mixing measurements published by ARGUS and CLEO [54–56] average to  $\chi_d = 0.182 \pm 0.015$ . Assuming no  $CP$  violation in mixing, the  $\Delta m_d$ ,  $1/\Gamma_d$ ,  $\Delta\Gamma_d/\Gamma_d$  and  $\chi_d$  results can be combined to yield the world average

$$\Delta m_d = 0.5068 \pm 0.0019 \text{ ps}^{-1} \quad (74.16)$$

or, equivalently,

$$\chi_d = 0.1849 \pm 0.0010. \quad (74.17)$$

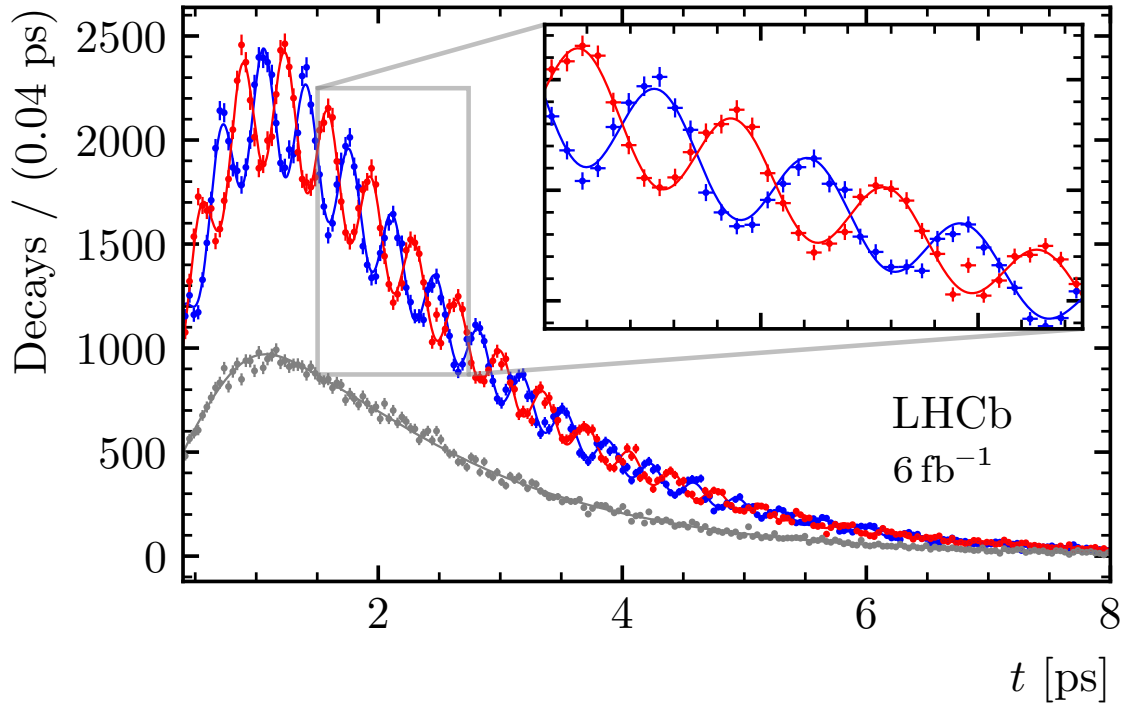
This  $\Delta m_d$  value provides an estimate of  $2|M_{12}|$ , and can be used with Eq. (74.6) to extract  $|V_{td}|$  within the Standard Model [57]. The main experimental uncertainties on the result come from  $m_t$  and  $\Delta m_d$ , but are still completely negligible with respect to the uncertainty due to the hadronic matrix element  $f_{B_d}\sqrt{B_{B_d}} = 225 \pm 9 \text{ MeV}$  [58] obtained from three-flavor lattice QCD calculations.

#### 74.4 $\Delta m_s$ and $\Delta\Gamma_s$ measurements

After many years of intense search at LEP and SLC,  $B_s^0$ - $\bar{B}_s^0$  oscillations were first observed in 2006 by CDF using  $1 \text{ fb}^{-1}$  of Tevatron Run II data [22]. LHCb then observed  $B_s^0$ - $\bar{B}_s^0$  oscillations independently with  $B_s^0 \rightarrow D_s^- \pi^+$  [43, 59],  $B_s^0 \rightarrow D_s^- \mu^+ \nu X$  [45] and  $B_s^0 \rightarrow J/\psi K^+ K^-$  [28] decays using up to  $3 \text{ fb}^{-1}$  of LHC Run 1 data. More recent measurements based on LHC Run 2 data have been published by CMS with  $B_s^0 \rightarrow J/\psi \phi$  decays [32], and by LHCb with  $B_s^0 \rightarrow D_s^- \pi^+ \pi^- \pi^+$  [60],  $B_s^0 \rightarrow D_s^- \pi^+$  [61] and  $B_s^0 \rightarrow J/\psi K^+ K^-$  [62] decays. Taking systematic correlations into account, the average [8] of all published measurements of  $\Delta m_s$  [22, 28, 32, 43, 45, 59–62] is

$$\Delta m_s = 17.766 \pm 0.004(\text{stat}) \pm 0.004(\text{syst}) \text{ ps}^{-1}, \quad (74.18)$$

—  $B_s^0 \rightarrow D_s^- \pi^+$     —  $\bar{B}_s^0 \rightarrow B_s^0 \rightarrow D_s^- \pi^+$     — Untagged



**Figure 74.2:** Proper decay-time distributions of background-subtracted  $B_s^0 \rightarrow D_s^- \pi^+$  decays tagged as unmixed (blue), tagged as mixed (red) or untagged (grey) in the Run 2 data of the LHCb experiment, displaying  $B_s^0-\bar{B}_s^0$  oscillations [61].

with an impressive precision dominated by the recent  $B_s^0 \rightarrow D_s^- \pi^+$  result [61] of Fig. 74.2.

The information on  $|V_{ts}|$  obtained in the framework of the Standard Model is hampered by the hadronic uncertainty, as in the  $B_d^0$  case. However, several uncertainties cancel in the frequency ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2, \quad (74.19)$$

where the SU(3) flavor-symmetry breaking factor  $\xi = (f_{B_s} \sqrt{B_{B_s}})/(f_{B_d} \sqrt{B_{B_d}})$  is obtained as  $1.206 \pm 0.017$  from a combination of three-flavor lattice QCD calculations [58] dominated by the results of Ref. [63], or as  $1.2014^{+0.0065}_{-0.0072}$  from QCD sum rules [64]. Using the measurements of Eqs. (74.16) and (74.18), one can extract

$$\left| \frac{V_{td}}{V_{ts}} \right| = \begin{cases} 0.2054 \pm 0.0004 \pm 0.0029 \text{ (lattice QCD)} \\ 0.2045 \pm 0.0004^{+0.0011}_{-0.0012} \text{ (QCD sum rules)} \end{cases}, \quad (74.20)$$

in good agreement with (but much more precise than) the value obtained from the ratio of the  $b \rightarrow d\gamma$  and  $b \rightarrow s\gamma$  transition rates observed at the  $B$  factories [57].

The CKM matrix can be constrained using experimental results on observables such as  $\Delta m_d$ ,  $\Delta m_s$ ,  $|V_{ub}/V_{cb}|$ ,  $\epsilon_K$ , and  $\sin(2\beta)$  together with theoretical inputs and unitarity conditions [57,65,66]. The constraint from our knowledge on the ratio  $\Delta m_s/\Delta m_d$  is more effective in limiting the position of the apex of the CKM unitarity triangle than the one obtained from the  $\Delta m_d$  measurements

alone, due to the reduced hadronic uncertainty in Eq. (74.19). We also note that the experimental value of  $\Delta m_s$  from Eq. (74.18) is consistent with the Standard Model predictions obtained from CKM fits where no experimental information on  $\Delta m_s$  is used, *e.g.*,  $17.26 \pm 0.58 \text{ ps}^{-1}$  [65] or  $17.05^{+0.75}_{-0.44} \text{ ps}^{-1}$  [66]. It is also consistent with the prediction  $18.23 \pm 0.62 \text{ ps}^{-1}$  from Ref. [7].

Information on  $\Delta\Gamma_s$  can be obtained from the study of the proper time distribution of untagged  $B_s^0$  samples [47]. In the case of an inclusive  $B_s^0$  selection [67], or a flavor-specific (semileptonic or hadronic)  $B_s^0$  decay selection [20, 68–70], both the short- and long-lived components are present, and the proper decay-time distribution is a superposition of two exponentials with decay constants  $\Gamma_{L,H} = \Gamma_s \pm \Delta\Gamma_s/2$ . In principle, this provides sensitivity to both  $\Gamma_s$  and  $(\Delta\Gamma_s/\Gamma_s)^2$ . Ignoring  $\Delta\Gamma_s$  and fitting for a single exponential leads to an estimate of  $1/\Gamma_s$  (called effective lifetime) with a relative bias proportional to  $(\Delta\Gamma_s/\Gamma_s)^2$ . An alternative approach, sensitive to first order in  $\Delta\Gamma_s/\Gamma_s$ , is to determine the effective lifetime of untagged  $B_s^0$  decays to pure  $CP$  eigenstates; measurements exist for  $B_s^0 \rightarrow D_s^+ D_s^-$  [69],  $B_s^0 \rightarrow K^+ K^-$  [70, 71],  $B_s^0 \rightarrow J/\psi\eta$  [72, 73],  $B_s^0 \rightarrow J/\psi f_0(980)$  [74],  $B_s^0 \rightarrow J/\psi\pi^+\pi^-$  [53, 75, 76],  $B_s^0 \rightarrow J/\psi K_S^0$  [77], and  $B_s^0 \rightarrow \mu^+\mu^-$  [78]. The extraction of  $1/\Gamma_s$  and  $\Delta\Gamma_s$  from such measurements, discussed in detail in Ref. [48], requires additional information in the form of theoretical assumptions or external inputs on weak phases and hadronic parameters. In what follows, we only use the effective lifetimes of decays to  $CP$ -even ( $D_s^+ D_s^-$ ,  $J/\psi\eta$ ) and  $CP$ -odd ( $J/\psi f_0(980)$ ,  $J/\psi\pi^+\pi^-$ ) final states where  $CP$  conservation can be assumed. In addition  $\Delta\Gamma_s$  can be extracted from the decay-time distributions of  $B_s^0$  decays to  $CP$ -even and  $CP$ -odd final states, as has been done by LHCb with  $B_s^0 \rightarrow J/\psi\eta'$  and  $B_s^0 \rightarrow J/\psi\pi^+\pi^-$ , respectively [79].

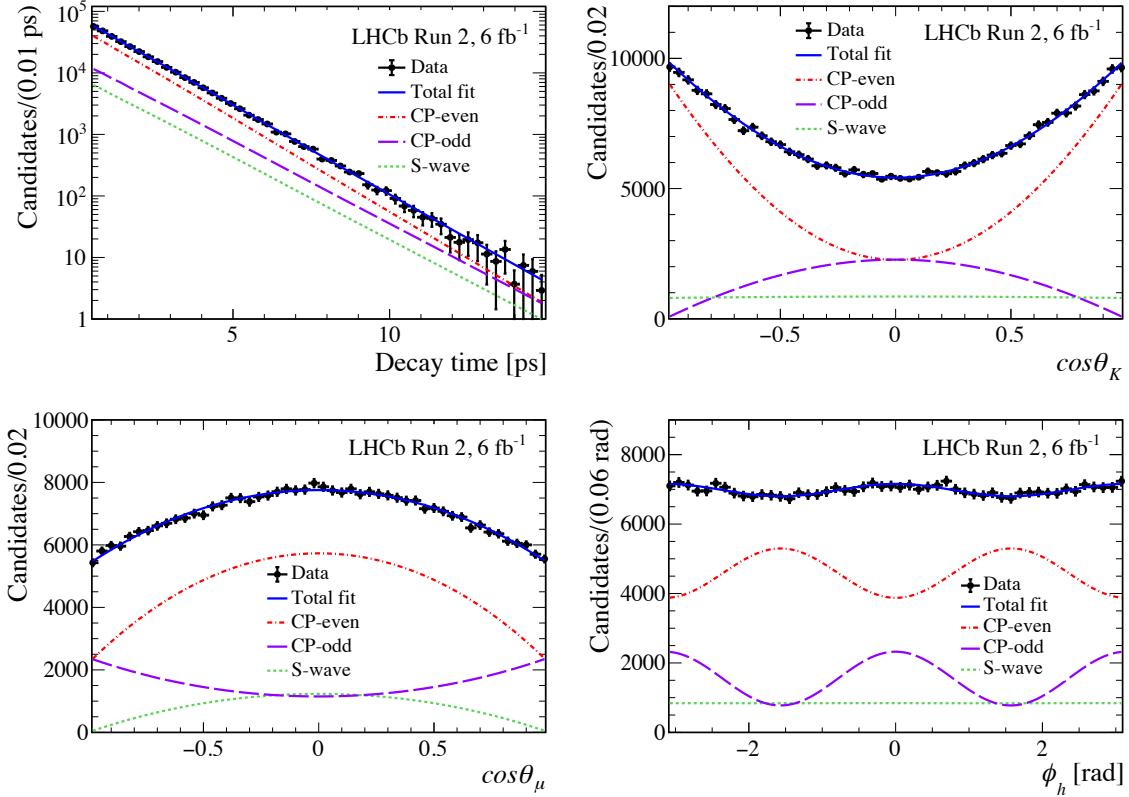
The best sensitivity to  $1/\Gamma_s$  and  $\Delta\Gamma_s$  is achieved by the time-dependent measurements of the  $B_s^0 \rightarrow J/\psi K^+ K^-$  (including  $B_s^0 \rightarrow J/\psi\phi$ ) and  $B_s^0 \rightarrow \psi(2S)\phi$  decay rates performed at CDF [80], DØ [81], ATLAS [31, 82], CMS [32, 83] and LHCb [28, 62, 84–86], where the  $CP$ -even and  $CP$ -odd amplitudes are separated statistically through a full angular analysis (see Fig. 74.3). The LHCb collaboration analyzed the  $B_s^0 \rightarrow J/\psi K^+ K^-$  decay considering that the  $K^+ K^-$  system can be in a P-wave or S-wave state, and measured the dependence of the strong phase difference between the P-wave and S-wave amplitudes as a function of the  $K^+ K^-$  invariant mass [28, 62, 87]; this allows the unambiguous determination of the sign of  $\Delta\Gamma_s$ , which is found to be positive. All these studies use both untagged and tagged  $B_s^0$  candidates and are optimized for the measurement of the phase  $\phi_s^{c\bar{c}s}$  that describes  $CP$  violation in the interference between  $B_s^0-\bar{B}_s^0$  mixing and decay in  $b \rightarrow c\bar{c}s$  transitions. The published  $B_s^0 \rightarrow J/\psi K^+ K^-$ ,  $J/\psi\phi$  and  $\psi(2S)\phi$  analyses [28, 31, 32, 62, 80–86] are combined in a multi-dimensional fit including all measured parameters and their correlations. To account for a tension in the time and angular parameters, scale factors are applied on the combined uncertainty of each parameter where a discrepancy arise. For example, the scale factors on the uncertainties of  $\Delta\Gamma_s$ ,  $\Gamma_s$  and  $\phi_s^{c\bar{c}s}$  are 1.65, 2.41 and 1.00, respectively. The averages are then further refined by applying constraints from the published lifetime measurements with flavor-specific [20, 68–70] and pure  $CP$  [53, 69, 72–76, 79] final states, to yield [8]

$$\Delta\Gamma_s = +0.0783 \pm 0.0035 \text{ ps}^{-1} \quad \text{and} \quad 1/\Gamma_s = 1.515 \pm 0.006 \text{ ps}, \quad (74.21)$$

or, equivalently,

$$1/\Gamma_{sL} = 1.430 \pm 0.006 \text{ ps} \quad \text{and} \quad 1/\Gamma_{sH} = 1.611 \pm 0.008 \text{ ps}, \quad (74.22)$$

in good agreement with the Standard Model predictions  $\Delta\Gamma_s^{\text{SM}} = +0.091 \pm 0.015 \text{ ps}^{-1}$  [7] and  $\Delta\Gamma_s^{\text{SM}} = +0.078 \pm 0.015 \text{ ps}^{-1}$  [9]. Estimates of  $\Delta\Gamma_s/\Gamma_s$  obtained from measurements of the  $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$  branching fractions are not included in the average, since they are based on the questionable [88] assumption that these decays account for all  $CP$ -even final states.



**Figure 74.3:** Distributions of the proper decay-time and of the three decay angles of background-subtracted  $B_s^0 \rightarrow J/\psi K^+ K^-$  decays reconstructed by LHCb in the vicinity of the  $\phi$  resonance [62]. The kaon pair is predominantly in a P-wave state (from the  $\phi$  decay) which can be CP-even or CP-odd, while a small S-wave state is CP-odd. The curves show the projections of a four-dimensional fit allowing the various components to be disentangled. The different lifetimes of the CP-even and CP-odd components are evident from the decay-time distribution.

From Eqs. (74.5), (74.18) and (74.21), one gets

$$\chi_s = 0.499313 \pm 0.000006. \quad (74.23)$$

### 74.5 $CP$ -violation studies

Evidence for  $CP$  violation in  $B_q^0-\bar{B}_q^0$  mixing has been searched for, both with flavor-specific and inclusive  $B_q^0$  decays, in samples where the initial flavor state is tagged, usually with a lepton from the other  $b$ -hadron in the event. In the case of semileptonic (or other flavor-specific) decays, where the final-state tag is also available, the following asymmetry [2]

$$\mathcal{A}_{\text{SL}}^q = \frac{N(\bar{B}_q^0(t) \rightarrow \ell^+ \nu_\ell X) - N(B_q^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)}{N(\bar{B}_q^0(t) \rightarrow \ell^+ \nu_\ell X) + N(B_q^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)} \simeq 1 - |q/p|_q^2 \quad (74.24)$$

has been measured either in time-integrated analyses at CLEO [56, 89], BaBar [90], CDF [91], DØ [92–94] and LHCb [95], or in time-dependent analyses at LEP [39, 96], BaBar [49, 97] and Belle [98]. In the inclusive case, also investigated at LEP [96, 99], no final-state tag is used, and the

asymmetry [100]

$$\begin{aligned} & \frac{N(\bar{B}_q^0(t) \rightarrow \text{all}) - N(B_q^0(t) \rightarrow \text{all})}{N(\bar{B}_q^0(t) \rightarrow \text{all}) + N(B_q^0(t) \rightarrow \text{all})} \\ & \simeq \mathcal{A}_{\text{SL}}^q \left[ \sin^2 \left( \frac{\Delta m_q t}{2} \right) - \frac{x_q}{2} \sin(\Delta m_q t) \right] \end{aligned} \quad (74.25)$$

must be measured as a function of the proper time to extract information on  $CP$  violation. In addition LHCb has studied the time dependence of the charge asymmetry of  $B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu X$  decays without tagging the initial state [101], which would be equal to

$$\frac{N(D^{(*)-} \mu^+ \nu_\mu X) - N(D^{(*)+} \mu^- \bar{\nu}_\mu X)}{N(D^{(*)-} \mu^+ \nu_\mu X) + N(D^{(*)+} \mu^- \bar{\nu}_\mu X)} = \mathcal{A}_{\text{SL}}^d \frac{1 - \cos(\Delta m_d t)}{2} \quad (74.26)$$

in absence of detection and production asymmetries.

The DØ collaboration measured a like-sign dimuon charge asymmetry in semileptonic  $b$  decays that deviates by  $2.8\sigma$  from the tiny Standard Model prediction and concluded, from a more refined analysis in bins of muon impact parameters, that the overall discrepancy is at the level of  $3.6\sigma$  [92]. In all other cases, asymmetries compatible with zero have been found, with a precision limited by the available statistics. Several of the analyses at high energy don't disentangle the  $B_d^0$  and  $B_s^0$  contributions, and either quote a mean asymmetry or a measurement of  $\mathcal{A}_{\text{SL}}^d$  assuming  $\mathcal{A}_{\text{SL}}^s = 0$ : we do not include these in the average. An exception is the dimuon DØ analysis [92], which separates the two contributions by exploiting their dependence on the muon impact parameter cut. The resulting measurements of  $\mathcal{A}_{\text{SL}}^d$  and  $\mathcal{A}_{\text{SL}}^s$  are then both compatible with the Standard Model. They are also correlated. We therefore perform a two-dimensional average of the measurements of Refs. [49, 56, 89, 90, 92–95, 97, 98, 101] and obtain [8]

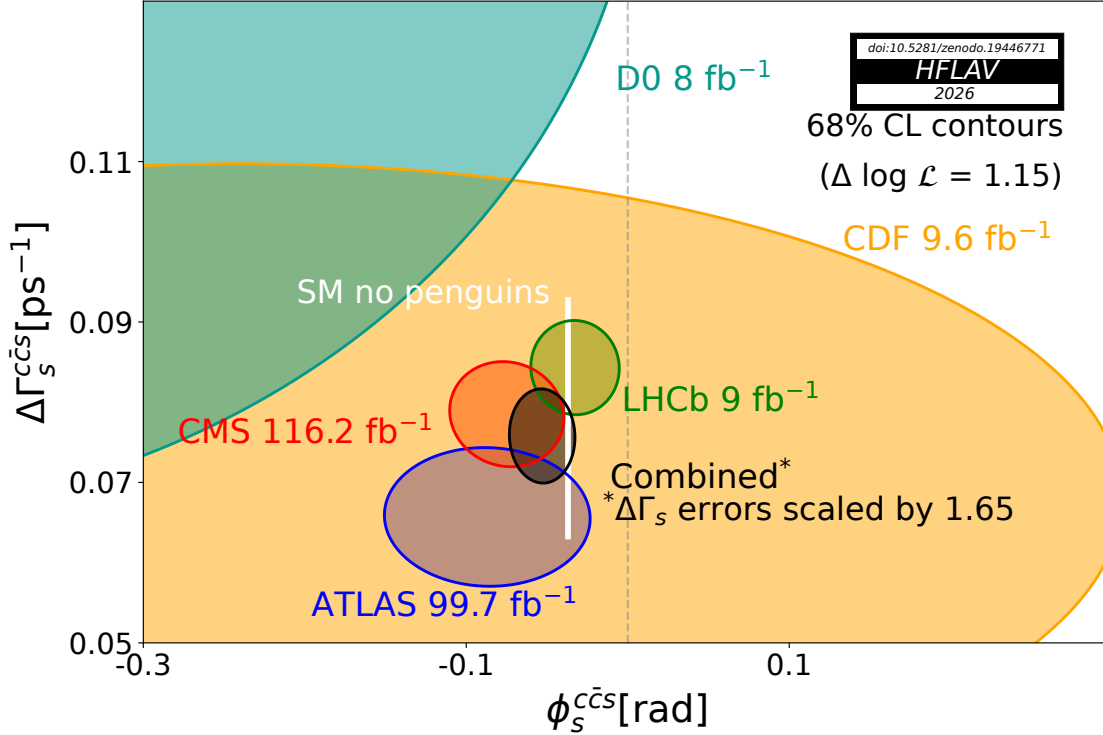
$$\mathcal{A}_{\text{SL}}^d = -0.0021 \pm 0.0017 \Leftrightarrow |q/p|_d = 1.0010 \pm 0.0008, \quad (74.27)$$

$$\mathcal{A}_{\text{SL}}^s = -0.0006 \pm 0.0028 \Leftrightarrow |q/p|_s = 1.0003 \pm 0.0014, \quad (74.28)$$

with a correlation coefficient of  $-0.054$  between  $\mathcal{A}_{\text{SL}}^d$  and  $\mathcal{A}_{\text{SL}}^s$ . These results show no evidence of  $CP$  violation and are compatible with the very small Standard Model predictions,  $\mathcal{A}_{\text{SL}}^{d,\text{SM}} = -(5.19 \pm 0.30) \times 10^{-4}$  and  $\mathcal{A}_{\text{SL}}^{s,\text{SM}} = +(2.27 \pm 0.13) \times 10^{-5}$  [9], but have insufficient precision yet to constrain the Standard Model.

$CP$  violation induced by  $B_s^0-\bar{B}_s^0$  mixing in  $b \rightarrow c\bar{c}s$  decays is controlled by the small weak phase  $\phi_s^{c\bar{c}s}$ . Measuring  $\phi_s^{c\bar{c}s}$  requires tagging the initial flavor of the decaying  $B_s^0$  meson. In addition to the previously mentioned  $B_s^0 \rightarrow J/\psi K^+ K^-$  (including  $B_s^0 \rightarrow J/\psi \phi$ ) and  $B_s^0 \rightarrow \psi(2S)\phi$  studies, the decay modes  $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$  (including  $B_s^0 \rightarrow J/\psi f_0(980)$ ) [76, 104] and  $B_s^0 \rightarrow D_s^+ D_s^-$  [102, 103] have also been analyzed by LHCb to measure  $\phi_s^{c\bar{c}s}$ , without the need for an angular analysis. The  $J/\psi \pi^+ \pi^-$  final state has been shown indeed to be (very close to) a pure  $CP$ -odd state [105]. In the  $B_s^0 \rightarrow J/\psi \phi$  and  $B_s^0 \rightarrow J/\psi K^+ K^-$  analyses,  $\phi_s^{c\bar{c}s}$  is obtained together with several other observables, including  $\Delta\Gamma_s$ ,  $\Gamma_s$ , the longitudinal and perpendicular  $\phi$  polarisation amplitudes, the S-wave amplitude, and strong phases. In order to account for all correlations, the full sets of measurements provided by the different analyses are combined in a multi-dimensional fit [8] of which  $\phi_s^{c\bar{c}s}$  is just one of the free parameters. As already mentioned the  $B_s^0 \rightarrow J/\psi \phi$  analyses of ATLAS, CMS and LHCb show a poor overall compatibility, mostly in the lifetime and angular parameters. Therefore, scale factors are applied on the combined uncertainty of each parameter where a discrepancy arises. For the parameters already in agreement, such as  $\phi_s^{c\bar{c}s}$ , no scale factor is applied. The combined result based on all published analyses [28, 31, 32, 62, 76, 80–86, 102–104] is

$$\phi_s^{c\bar{c}s} = -0.053 \pm 0.013. \quad (74.29)$$



**Figure 74.4:** 68% CL contours in the  $(\phi_s^{ccs}, \Delta\Gamma_s)$  plane, showing all measurements from CDF [80], DØ [81], ATLAS [31,82], CMS [32,83] and LHCb [28,62,76,84–86,102–104] using  $B_s^0$  decays governed by the  $b \rightarrow c\bar{c}s$  transition. Their average [8] is represented as the black ellipse, where the combined uncertainty on  $\Delta\Gamma_s$  has been multiplied by 1.65. The very thin white rectangle represents the Standard Model predictions of  $-2\beta_s$  [65,66] and  $\Delta\Gamma_s$  [9].

A two-dimensional projection of the overall situation in the  $(\phi_s^{ccs}, \Delta\Gamma_s)$  plane is shown in Fig. 74.4. The experimental determination of  $\phi_s^{ccs}$  is still statistically limited. It is consistent with the Standard Model prediction, which is equal to  $-2\beta_s = -2 \arg(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)) = -0.0367 \pm 0.0010$  [65] or  $-0.0377 \pm 0.0006$  [66] assuming negligible Penguin pollution.

$CP$  violation induced by  $B_s^0-\bar{B}_s^0$  mixing in  $b \rightarrow s\bar{s}s$  decays is controlled by the weak phase  $\phi_s^{s\bar{s}s}$ , which is an observable different from  $\phi_s^{ccs}$ . In the Standard Model, such pure gluonic Penguin decays have an amplitude with a CKM phase that cancels that of the mixing amplitude, and hence  $\phi_s^{s\bar{s}s}$  is expected to be essentially zero. LHCb has performed a flavor-tagged time-dependent angular analysis of  $B_s^0 \rightarrow \phi\phi$  decays [29,106], similar to that of  $B_s^0 \rightarrow J/\psi\phi$  decays, and measured

$$\phi_s^{s\bar{s}s} = -0.074 \pm 0.069, \quad (74.30)$$

in agreement with the Standard Model prediction.

## 74.6 Summary

$B^0-\bar{B}^0$  mixing has been and still is a field of intense study. The mass differences in the  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  systems are known to relative precisions of 0.37% and 0.03%, respectively. The non-zero decay-width difference in the  $B_s^0-\bar{B}_s^0$  system is well established, with a relative difference of  $\Delta\Gamma_s/\Gamma_s = (11.8 \pm 0.6)\%$ , meaning that the heavy state of the  $B_s^0-\bar{B}_s^0$  system lives  $\sim 13\%$  longer than the light state. In contrast, the relative decay-width difference in the  $B_d^0-\bar{B}_d^0$  system,

$\Delta\Gamma_d/\Gamma_d = (0.5 \pm 1.0)\%$ , is still consistent with zero.  $CP$  violation in  $B_d^0-\bar{B}_d^0$  or  $B_s^0-\bar{B}_s^0$  mixing has not been observed yet, with precisions on the semileptonic asymmetries below 0.3%.  $CP$  violation induced by  $B_s^0-\bar{B}_s^0$  mixing in  $B_s^0$  decays has not yet been observed either, with uncertainties on the  $\phi_s^{ccs}$  and  $\phi_s^{sss}$  phases of 13 mrad and 69 mrad, respectively. All observations so far remain consistent with the Standard Model expectations.

However, the measurements where New Physics might show up are still statistically limited. More results are awaited from the LHC experiments and Belle II, with promising prospects for the investigation of the  $CP$ -violating phase  $\arg(-M_{12}/\Gamma_{12})$  and improved determination of the  $\phi_s^{ccs}$  and  $\phi_s^{sss}$  phases.

Mixing studies have clearly reached the stage of precision measurements, where much effort is needed, both on the experimental and theoretical sides, in particular to further reduce the hadronic uncertainties of lattice QCD calculations. In the long term, a stringent check of the consistency of the  $B_d^0$  and  $B_s^0$  mixing amplitudes (magnitudes and phases) with all other measured flavor-physics observables will be possible within the Standard Model, leading to very tight limits on (or otherwise a long-awaited surprise about) New Physics.

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