

70. D^0 - \bar{D}^0 Mixing

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The formalism for D^0 - \bar{D}^0 mixing is closely related to that for CP violation; for further details on the latter, see the note “ CP Violation in the Quark Sector” in this *Review*. The time evolution of the D^0 - \bar{D}^0 system is described by the Schrödinger equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \right) \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}, \quad (70.1)$$

where the \mathbf{M} and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance requires that $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of \mathbf{M} and $\mathbf{\Gamma}$ are referred to as the dispersive and absorptive parts, respectively, of the mixing. The mass eigenstates D_1 and D_2 of the Hamiltonian $\mathbf{M} - i\mathbf{\Gamma}/2$ are defined as

$$|D_{1,2}\rangle \equiv p|D^0\rangle \pm q|\bar{D}^0\rangle, \quad (70.2)$$

where normalization imposes $|p|^2 + |q|^2 = 1$. Equation (70.1) implies that a charm meson created as a D^0 will time-evolve to a \bar{D}^0 , and vice-versa. As shown in Eqs. (70.11) and (70.13) below, the probability for $D^0 \rightarrow \bar{D}^0 \propto |q/p|^2$, whereas the probability for $\bar{D}^0 \rightarrow D^0 \propto |p/q|^2$. Thus, if $|q/p| \neq 1$, CP is violated in mixing. If $\text{Arg}(q/p) \neq 0$ or π , CP is violated in the interference between a mixed decay amplitude and a direct decay amplitude. These two types of CP violation are collectively referred to as *indirect* CP violation.

As the absolute phases of $|D^0\rangle$ and $|\bar{D}^0\rangle$ are arbitrary, the relative phase ξ between them is unphysical. This relative phase must appear in a CP transformation: $CP|D^0\rangle = e^{i\xi}|\bar{D}^0\rangle$ and $CP|\bar{D}^0\rangle = e^{-i\xi}|D^0\rangle$, resulting in $(CP)^2|D^0\rangle = |D^0\rangle$ and $(CP)^2|\bar{D}^0\rangle = |\bar{D}^0\rangle$. For definiteness we choose a phase convention $CP|D^0\rangle = -|\bar{D}^0\rangle$, which fixes $\xi = \pi$. In addition, for $\text{Arg}(q/p)$ to have physical meaning it must be defined relative to a reference phase, i.e., only phase *differences* are physical. In Ref. [1], the reference phase is taken to be that of the $\Delta(U\text{-spin}) = 2$ component of $\langle \bar{D}^0 | \Gamma | D^0 \rangle$; this is equivalent to defining this component of Γ_{12} to be real and positive, and we retain this definition here. With these conventions, if $p = \pm q$ then D_1, D_2 are CP eigenstates, and there is no indirect CP violation ($|q/p| = 1$ and $\text{Arg}(q/p) = 0$ or π).

The eigenvalues of the Hamiltonian $\mathbf{M} - i\mathbf{\Gamma}/2$ are

$$\omega_{1,2} = \left(M - \frac{i}{2} \Gamma \right) \pm \frac{q}{p} \left(M_{12} - \frac{i}{2} \Gamma_{12} \right) \equiv m_{1,2} - \frac{i}{2} \Gamma_{1,2}, \quad (70.3)$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are defined to be real and thus correspond to the masses and decay widths, respectively, of the eigenstates $D_{1,2}$. As the traces $M_{11} + M_{22} = 2M$ and $\Gamma_{11} + \Gamma_{22} = 2\Gamma$ are unchanged by diagonalizing \mathbf{M} and $\mathbf{\Gamma}$, M must equal the mean value $(M_1 + M_2)/2$ and Γ must equal the mean value $(\Gamma_1 + \Gamma_2)/2$. Solving for the eigenstates of the Hamiltonian yields

$$\left(\frac{q}{p} \right)^2 = \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}. \quad (70.4)$$

If $\text{Arg}(M_{12}) = \text{Arg}(\Gamma_{12})$, then $|q/p| = 1$ and CP is conserved in mixing. In the more restrictive case that M_{12} and Γ_{12} are real, Eq. (70.4) implies that $\text{Arg}(q/p) = 0$ or π and there is no indirect CP violation. In this case, Eq. (70.3) implies that the differences in eigenvalues $\Delta m \equiv m_2 - m_1 = 2M_{12}$

and $\Delta\Gamma \equiv \Gamma_2 - \Gamma_1 = \Gamma_{12}$. The magnitudes and signs of Δm and $\Delta\Gamma$ are difficult to predict from theory and must be determined experimentally.

It is advantageous to define dimensionless mixing parameters x and y as

$$x \equiv \frac{\Delta m}{\Gamma} \quad y \equiv \frac{\Delta\Gamma}{2\Gamma}. \quad (70.5)$$

These parameters are measured in several ways; the most precise values are obtained by measuring the time dependence of D^0 decays. As an alternative to parameters x, y , one can define mixing parameters [2]

$$x_{12} \equiv \frac{2|M_{12}|}{\Gamma} \quad y_{12} \equiv \frac{|\Gamma_{12}|}{\Gamma}. \quad (70.6)$$

As shown above, CP violation in mixing arises from $\text{Arg}(M_{12}) \neq \text{Arg}(\Gamma_{12})$, and CP violation due to interference arises from either $\text{Arg}(M_{12}) \equiv \phi^M \neq 0, \pi$ or $\text{Arg}(\Gamma_{12}) \equiv \phi^F \neq 0, \pi$. Defining $\phi \equiv \text{Arg}(q/p) + n\pi$, the conversion between parameters $(x, y, |q/p|, \phi)$ and equivalent parameters $(x_{12}, y_{12}, \phi^M, \phi^F)$ is [1, 3]

$$x^2 = \frac{x_{12}^2 - y_{12}^2 + \sqrt{(x_{12}^2 + y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2 \phi_{12}}}{2} \quad (70.7)$$

$$y^2 = \frac{y_{12}^2 - x_{12}^2 + \sqrt{(x_{12}^2 + y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2 \phi_{12}}}{2} \quad (70.8)$$

$$\left| \frac{q}{p} \right|^2 = \sqrt{\frac{x_{12}^2 + y_{12}^2 + 2x_{12}y_{12} \sin \phi_{12}}{x_{12}^2 + y_{12}^2 - 2x_{12}y_{12} \sin \phi_{12}}} \quad (70.9)$$

$$\tan(2\phi) = -\frac{x_{12}^2 \sin(2\phi^M) + y_{12}^2 \sin(2\phi^F)}{x_{12}^2 \cos(2\phi^M) + y_{12}^2 \cos(2\phi^F)}, \quad (70.10)$$

where $\phi_{12} \equiv \phi^M - \phi^F$. If CP is conserved in mixing ($|q/p| = 1$), then $\phi_{12} = 0, \pi$ and $|x| = x_{12}$, $|y| = y_{12}$. In this case, Eq. (70.10) implies $\phi = -\phi^{M,F}$ modulo π . It can be shown that the product $xy = x_{12}y_{12} \cos \phi_{12}$, and thus the fact that x and y are measured to be positive implies ϕ_{12} is close to zero rather than π . It can also be shown that the parameter $y_{CP}(K^+K^-, \pi^+\pi^-) \approx y_{12} \cos \phi^F$, and thus the fact that y_{CP} is measured to be positive implies ϕ^F (and hence ϕ^M) is close to zero rather than π .

To measure mixing, the initial flavor of the D^0 or \bar{D}^0 when produced must be determined. The most common method used to identify the initial flavor is to reconstruct $D^{*+} \rightarrow D^0\pi^+$ or $D^{*-} \rightarrow \bar{D}^0\pi^-$ decays; the charge of the accompanying pion (which has low momentum in the lab frame and is often referred to as the ‘‘soft’’ pion) determines the flavor of the neutral D . BABAR and LHCb have also identified the flavor of the neutral D by reconstructing the semileptonic decays $B^+ \rightarrow \bar{D}^0 X\ell^+\nu$, $B^0 \rightarrow D^{*-} X\ell^+\nu$, $B^- \rightarrow D^0 X\ell^-\bar{\nu}$, and $\bar{B}^0 \rightarrow D^{*+} X\ell^-\bar{\nu}$;¹ in this case the charge of the accompanying lepton determines (or ‘‘tags’’) the D flavor. Both experiments have used these tags together to select ‘‘double-tagged’’ $B^0 \rightarrow D^{*-} X\ell^+\nu$, $D^{*-} \rightarrow \bar{D}^0\pi^-$ decays, which have especially high purity. At e^+e^- collider experiments such as Belle, Belle II, BABAR, and BESIII, the D flavor can also be determined by reconstructing a flavor-specific D decay on the ‘‘opposite side’’ of an event, i.e., recoiling against the signal-side D decay.

At BESIII, where $D\bar{D}$ pairs are produced near their threshold via $e^+e^- \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0$, there is little background and the purity of opposite-side tagging is equivalent to that achieved

¹Charge-conjugate modes are implicitly included throughout this Review, unless noted otherwise.

using $D^{*\pm}$ decays. However, BESIII operates at a symmetric e^+e^- collider, and the $D\bar{D}$ pairs are produced almost at rest in the lab frame. As a consequence, the D 's do not travel any appreciable distance before decaying, and time-dependent analyses are not possible. To overcome this, measurements of mixing at BESIII utilize the quantum coherence of the initial $\psi(3770) \rightarrow D^0\bar{D}^0$ state and time-integrated measurements [4–8].

70.1 Time-Dependent Analyses

Starting from a pure $|D^0\rangle$ state, after a time t there will exist a mixture of states:

$$|D^0(t)\rangle = g_+(t)|D^0\rangle + \left(\frac{q}{p}\right)g_-(t)|\bar{D}^0\rangle, \quad (70.11)$$

where

$$g_{\pm}(t) = \frac{1}{2} \left(e^{-i\omega_1 t} \pm e^{-i\omega_2 t} \right) \quad (70.12)$$

and $\omega_{1,2}$ are defined in Eq. (70.3). Similarly, a pure $|\bar{D}^0\rangle$ state will evolve to a mixture of states

$$|\bar{D}^0(t)\rangle = g_+(t)|\bar{D}^0\rangle + \left(\frac{p}{q}\right)g_-(t)|D^0\rangle. \quad (70.13)$$

The respective decay rates to a final state f are

$$r_f(t) \equiv \left| \langle f|H|D^0(t)\rangle \right|^2 = \left| g_+(t)\langle f|H|D^0\rangle + \left(\frac{q}{p}\right)g_-(t)\langle f|H|\bar{D}^0\rangle \right|^2 \quad (70.14)$$

$$\bar{r}_f(t) \equiv \left| \langle f|H|\bar{D}^0(t)\rangle \right|^2 = \left| g_+(t)\langle f|H|\bar{D}^0\rangle + \left(\frac{p}{q}\right)g_-(t)\langle f|H|D^0\rangle \right|^2. \quad (70.15)$$

In these expressions we have neglected phase-space factors, which divide out in ratios of D^0 and \bar{D}^0 decay rates to common final states. Denoting the decay amplitudes of pure D^0 and \bar{D}^0 states as $A_f \equiv \langle f|H|D^0\rangle$, $\bar{A}_f \equiv \langle f|H|\bar{D}^0\rangle$, $A_{\bar{f}} \equiv \langle \bar{f}|H|D^0\rangle$, and $\bar{A}_{\bar{f}} \equiv \langle \bar{f}|H|\bar{D}^0\rangle$, and defining the combinations of factors

$$\lambda_f \equiv \frac{q\bar{A}_f}{pA_f} \quad \lambda_{\bar{f}} \equiv \frac{q\bar{A}_{\bar{f}}}{pA_{\bar{f}}}, \quad (70.16)$$

one obtains

$$r_f(t) = |A_f|^2 \left[|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\text{Re}(\lambda_f g_+^*(t) g_-(t)) \right] \quad (70.17)$$

$$= \frac{|A_f|^2}{2} e^{-\Gamma t} \left[(1 + |\lambda_f|^2) \cosh(y\Gamma t) + 2\text{Re}(\lambda_f) \sinh(y\Gamma t) + (1 - |\lambda_f|^2) \cos(x\Gamma t) - 2\text{Im}(\lambda_f) \sin(x\Gamma t) \right] \quad (70.18)$$

$$\bar{r}_f(t) = |\bar{A}_f|^2 \left[|g_+(t)|^2 + |\lambda_f|^{-2} |g_-(t)|^2 + 2\text{Re}(\lambda_f^{-1} g_+^*(t) g_-(t)) \right] \quad (70.19)$$

$$= \frac{|\bar{A}_f|^2}{2} e^{-\Gamma t} \left[(1 + |\lambda_f|^{-2}) \cosh(y\Gamma t) + 2\text{Re}(\lambda_f^{-1}) \sinh(y\Gamma t) + (1 - |\lambda_f|^{-2}) \cos(x\Gamma t) - 2\text{Im}(\lambda_f^{-1}) \sin(x\Gamma t) \right]. \quad (70.20)$$

The rates $r_{\bar{f}}$ and $\bar{r}_{\bar{f}}$ are identical to Eqs. (70.18) and (70.20), respectively, except that f is replaced by \bar{f} . We note that a change in convention for the relative phase between D^0 and \bar{D}^0 cancels between q/p and \bar{A}_f/A_f or $\bar{A}_{\bar{f}}/A_{\bar{f}}$, leaving λ_f and $\lambda_{\bar{f}}$ unchanged.

As $x, y \ll 1$ and experiments typically measure t in the range $(1-8)/\Gamma$, the cosh, sinh, cosine, and sine functions can be expanded to second order in x and y . One obtains:

$$r_f(t) \approx |A_f|^2 e^{-\Gamma t} \left\{ 1 + [y \operatorname{Re}(\lambda_f) - x \operatorname{Im}(\lambda_f)](\Gamma t) + |\lambda_f|^2 \left(\frac{x^2 + y^2}{4} \right) (\Gamma t)^2 + \left(\frac{y^2 - x^2}{4} \right) (\Gamma t)^2 \right\} \quad (70.21)$$

$$\bar{r}_f(t) \approx |\bar{A}_f|^2 e^{-\Gamma t} \left\{ 1 + \left[y \operatorname{Re}\left(\frac{1}{\lambda_f}\right) - x \operatorname{Im}\left(\frac{1}{\lambda_f}\right) \right] (\Gamma t) + \frac{1}{|\lambda_f|^2} \left(\frac{x^2 + y^2}{4} \right) (\Gamma t)^2 + \left(\frac{y^2 - x^2}{4} \right) (\Gamma t)^2 \right\}. \quad (70.22)$$

For $r_{\bar{f}}$ and $\bar{r}_{\bar{f}}$, A_f , \bar{A}_f , and λ_f are replaced by $A_{\bar{f}}$, $\bar{A}_{\bar{f}}$, and $\lambda_{\bar{f}}$. In the sections below we discuss various final states f ; our convention is that amplitudes A_f and $\bar{A}_{\bar{f}}$ are Cabibbo-favored (CF), whereas \bar{A}_f and $A_{\bar{f}}$ are Cabibbo-suppressed. With this convention, $|\lambda_f| \ll 1$ and $|1/\lambda_{\bar{f}}| \ll 1$. For multibody final states, Eqs. (70.21) and (70.22) apply separately to each point in phase-space.

70.2 Semileptonic decays

Consider the semileptonic decay $D^0 \rightarrow K^- \ell^+ \nu$. For this final state, $\bar{A}_f = A_{\bar{f}} = 0$ to excellent approximation.² Thus, the final state f is reached from a pure \bar{D}^0 state at $t = 0$ only via mixing. The parameters $\lambda_f = 1/\lambda_{\bar{f}} = 0$, and Eqs. (70.21) and (70.22) become

$$r_f(t) = |A_f|^2 e^{-\Gamma t} \quad \bar{r}_f(t) = |A_f|^2 e^{-\Gamma t} \left| \frac{p}{q} \right|^2 \left(\frac{x^2 + y^2}{4} \right) (\Gamma t)^2, \quad (70.23)$$

Table 70.1: Results for $R_M = (x^2 + y^2)/2$ in D^0 semileptonic decays. The HFLAV average assumes statistical and systematic uncertainties are uncorrelated. When a single uncertainty is listed, it corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been superseded and are not included in the HFLAV average.

Year	Experiment	Final state(s)	$R_M (\times 10^{-3})$	90% C.L. ($\times 10^{-3}$)
2008	Belle (492 fb^{-1}) [9]	$K^{(*)+} e^- \bar{\nu}_e$	$0.13 \pm 0.22 \pm 0.20$	< 0.61
2007	BABAR (344 fb^{-1}) [10]	$K^{(*)+} e^- \bar{\nu}_e$	$0.04^{+0.70}_{-0.60}$	$(-1.3, 1.2)$
2005	CLEO (9.0 fb^{-1}) [11]	$K^{(*)+} e^- \bar{\nu}_e$	$1.6 \pm 2.9 \pm 2.9$	< 7.8
1996	E791 (2×10^{10} evts) [12]	$K^+ \ell^- \bar{\nu}_\ell$	$1.1^{+3.0}_{-2.7} {}^{+0.0}_{-0.1}$	< 5.0
HFLAV Average [13]			0.130 ± 0.269	
2005*	Belle (253 fb^{-1}) [14]	$K^{(*)+} e^- \bar{\nu}_e$	$0.02 \pm 0.47 \pm 0.14$	< 1.0
2004*	BABAR (87 fb^{-1}) [15]	$K^{(*)+} e^- \bar{\nu}_e$	$2.3 \pm 1.2 \pm 0.4$	< 4.2

where the term proportional to $(y^2 - x^2)(\Gamma t)^2$ in $r_f(t)$ is negligible and thus dropped. For $r_{\bar{f}}(t)$ and $\bar{r}_{\bar{f}}(t)$, the former equals the second expression in Eq. (70.23) with $|q/p|^2$ replacing $|p/q|^2$, and the latter equals the first expression. In the Standard Model, CP violation in charm mixing is tiny and $|q/p| \approx 1$. Assuming such CP violation is negligible, the time-integrated mixed decay rate relative to the time-integrated unmixed decay rate is

$$\frac{\int_0^\infty \bar{r}_f(t) dt}{\int_0^\infty r_f(t) dt} = \frac{\int_0^\infty r_{\bar{f}}(t) dt}{\int_0^\infty \bar{r}_{\bar{f}}(t) dt} = \frac{x^2 + y^2}{2} \equiv R_M. \quad (70.24)$$

²There exists a doubly Cabibbo-suppressed amplitude in which the c and \bar{u} quarks exchange a W , and the resulting d quark from the c decays semileptonically. We neglect this higher-order process.

Measurements of R_M from semileptonic decays are listed in Table 70.1. The world average from the Heavy Flavor Averaging Group (HFLAV) [16] is $R_M = (1.30 \pm 2.69) \times 10^{-4}$.

70.3 Hadronic decays to non- CP eigenstates

Consider the hadronic decay $D^0 \rightarrow K^- \pi^+$, i.e., \bar{A}_f and A_f are doubly Cabibbo-suppressed (DCS). Because these CF and DCS decays are dominated by tree-level amplitudes involving only the first two quark generations, direct CP violation is expected to be very small. The ratios of decay amplitudes can be written

$$\frac{\bar{A}_f}{A_f} = \sqrt{R_D^-} e^{-i\delta_f} \quad \frac{A_f}{\bar{A}_f} = \sqrt{R_D^+} e^{-i\delta_f}, \quad (70.25)$$

where R_D^- and R_D^+ are the ratios of the DCS decay rate to the CF decay rate: $R_D^- \equiv |\bar{A}_f/A_f|^2$ and $R_D^+ \equiv |A_f/\bar{A}_f|^2$. A relative minus sign resulting from the weak phase difference between $V_{cd}^* V_{us}$ (for \bar{A}_f) and $V_{cs}^* V_{ud}$ (for A_f) is canceled by the phase difference π between D^0 and \bar{D}^0 resulting from our convention $CP|D^0\rangle = -|\bar{D}^0\rangle$. From the CKM matrix elements, one estimates $R_D^-, R_D^+ \approx \tan^4 \theta_c$, where θ_c is the Cabibbo angle. The parameter δ_f is the strong phase difference between CF and DCS amplitudes: $\delta \equiv \text{Arg}(A_f/\bar{A}_f) = \text{Arg}(\bar{A}_f/A_f)$. The amplitudes A_f and \bar{A}_f can in principle involve higher-order (or sub-leading) diagrams with a different weak phase than the leading diagram. The phase differences φ_f would depend on the final state and contribute factors $e^{i\varphi_f}$ to Eq. (70.25). However, such amplitudes are highly suppressed and can be safely neglected here; this is referred to as ‘‘approximate universality.’’ Eq. (70.25) implies

$$\lambda_f = \sqrt{R_D^-} \left| \frac{q}{p} \right| e^{-i(\delta_f - \phi)} \quad (\lambda_f)^{-1} = \sqrt{R_D^+} \left| \frac{p}{q} \right| e^{-i(\delta_f + \phi)}, \quad (70.26)$$

where $\phi = \text{Arg}(q/p)$. For convenience, we define the mean decay rate $R_D \equiv (R_D^+ + R_D^-)/2$ and the decay rate asymmetry $A_D \equiv (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$. With these definitions, the rates for the mixed (‘‘wrong-sign’’) decays $D^0 \rightarrow K^+ \pi^-$ and $\bar{D}^0 \rightarrow K^- \pi^+$ are [17, 18]:

$$r_{\bar{f}}(t) = |\bar{A}_{\bar{f}}|^2 e^{-\Gamma t} \left[R_D(1 + A_D) + \sqrt{R_D(1 + A_D)} \left| \frac{q}{p} \right| y'_+(\Gamma t) + \left| \frac{q}{p} \right|^2 \frac{(x'_+{}^2 + y'_+{}^2)}{4} (\Gamma t)^2 \right] \quad (70.27)$$

$$\bar{r}_f(t) = |A_f|^2 e^{-\Gamma t} \left[R_D(1 - A_D) + \sqrt{R_D(1 - A_D)} \left| \frac{p}{q} \right| y'_-(\Gamma t) + \left| \frac{p}{q} \right|^2 \frac{(x'_-{}^2 + y'_-{}^2)}{4} (\Gamma t)^2 \right], \quad (70.28)$$

where

$$x'_\pm = x' \cos \phi \pm y' \sin \phi \quad (70.29)$$

$$y'_\pm = y' \cos \phi \mp x' \sin \phi, \quad (70.30)$$

and

$$x' \equiv x \cos \delta_f + y \sin \delta_f \quad (70.31)$$

$$y' \equiv y \cos \delta_f - x \sin \delta_f. \quad (70.32)$$

The parameters x', y' are obtained from x, y via a rotation by the strong phase δ ; these are often referred to as ‘‘strong-phase-rotated’’ mixing parameters. The parameters x'_\pm, y'_\pm are obtained from x', y' via a subsequent rotation by the weak phase $+\phi$ for D^0 decays and $-\phi$ for \bar{D}^0 decays. Note that $x'_+{}^2 + y'_+{}^2 = x'_-{}^2 + y'_-{}^2 = x'^2 + y'^2 = x^2 + y^2$. In Eqs. (70.27) and (70.28), a fourth term

proportional to $R_D(1 \pm A_D)(x_{\pm}^2 - y_{\pm}^2)(\Gamma t)^2/4$ has been dropped, as it is negligible relative to the other terms for the range of decay times measured by experiments.

Comparing Eqs. (70.27) and (70.28), one sees that $r_{\bar{f}}(t) \neq \bar{r}_f(t)$ if either $A_D \neq 0$, $|q/p| \neq 1$, or $\phi \neq 0$. The first inequality corresponds to CP violation in the decay amplitudes ($R_D^+ \neq R_D^-$), which is referred to as *direct CP* violation. The second inequality corresponds to CP violation in mixing, and the third inequality corresponds to CP violation due to interference between a mixed decay amplitude and an unmixed amplitude. Whereas CP violation in decay amplitudes is parameterized by A_D , CP violation in mixing is often parameterized by $A_M \equiv (|q/p| - |p/q|)/(|q/p| + |p/q|)$.

In the limit of CP conservation, $A_D = 0$, $|q/p| = 1$, and $\phi = 0$. In this case,

$$r_{\bar{f}}(t) = \bar{r}_f(t) = |A_f|^2 e^{-\Gamma t} \left[R_D + \sqrt{R_D} y'(\Gamma t) + \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right]. \quad (70.33)$$

The time-integrated mixed decay rate relative to the time-integrated unmixed decay rate is

$$R = \frac{\int_0^{\infty} r_{\bar{f}}(t) dt}{\int_0^{\infty} |A_f|^2 e^{-\Gamma t} dt} = R_D + \sqrt{R_D} y' + \frac{x'^2 + y'^2}{2}. \quad (70.34)$$

The ratio R is straightforward to measure, as there is no decay-time dependence. Parameters R_D, x'^2, y' are measured by assuming CP conservation and fitting to the decay-time distribution of wrong-sign ($D^0 + \bar{D}^0$) decays. Experiments allow for CP violation and measure $x'^2, y', A_D, |q/p|, \phi$ by separately fitting the decay-time distributions of wrong-sign $D^0 \rightarrow K^+ \pi^-$ and $\bar{D}^0 \rightarrow K^- \pi^+$ decays. Results for R, R_D , and A_D are listed in Table 70.2, and results for x'^2 and y' are listed in Table 70.3.

The LHCb experiment recently analyzed all Runs 1+2 data with a D^* -tag, fitting to the ratios $r_{\bar{f}}(t)/\bar{r}_{\bar{f}}(t) \equiv R_{K\pi}^+(t)$ and $\bar{r}_f(t)/r_f(t) \equiv R_{K\pi}^-(t)$ in bins of decay time [19]. The fitted functions are parameterized as

$$R_{K\pi}^{\pm}(t) = R_D(1 \pm A_D) + \sqrt{R_D(1 \pm A_D)}(c_{K\pi} \pm \Delta c_{K\pi})(\Gamma t) + (c'_{K\pi} \pm \Delta c'_{K\pi})(\Gamma t)^2. \quad (70.35)$$

Fit results for parameters $c_{K\pi}, \Delta c_{K\pi}, c'_{K\pi}$, and $\Delta c'_{K\pi}$ are listed also in Table 70.3.

Extraction of the mixing parameters x and y from measurements of x' and y' requires knowledge of the strong phase difference $\delta_{K\pi}$. This can be determined from the decay rates of $D_{\pm} \rightarrow K^+ \pi^-$, where D_+ (D_-) denotes the CP -even (CP -odd) eigenstate. Since $|D_{\pm}\rangle = (|D^0\rangle \mp |\bar{D}^0\rangle)/\sqrt{2}$,

$$\sqrt{2} A(D_{\pm} \rightarrow K^+ \pi^-) = A(D^0 \rightarrow K^+ \pi^-) \mp A(\bar{D}^0 \rightarrow K^+ \pi^-). \quad (70.36)$$

Squaring this amplitude and using Eq. (70.25) yields the relation

$$\cos \delta_{K\pi} = \frac{|A(D_- \rightarrow K^+ \pi^-)|^2 - |A(D_+ \rightarrow K^+ \pi^-)|^2}{2|A(D^0 \rightarrow K^+ \pi^-)||A(\bar{D}^0 \rightarrow K^+ \pi^-)|}. \quad (70.37)$$

Measuring the right-hand side is possible if one can identify pure D_+ , D_- , D^0 , and \bar{D}^0 initial states. This is accomplished at CLEOc and BESIII utilizing the processes $e^+e^- \rightarrow \psi(3770) \rightarrow \bar{D}^0 D^0 \rightarrow (f_{CP})(K^+ \pi^-)$, or $\psi(3770) \rightarrow \bar{D}^0 D^0 \rightarrow (f_{\bar{D}^0})(K^+ \pi^-)$, where f_{CP} denotes a CP -specific final state, and $f_{\bar{D}^0}$ denotes a \bar{D}^0 -flavor-specific final state. In the first case, quantum coherence and CP symmetry ensures that the $K^+ \pi^-$ state originates from a neutral D with CP opposite that of f_{CP} . In the second case, when the \bar{D}^0 decays, the opposite side must be D^0 . However, it

Table 70.2: Results for R , R_D , and A_D measured using $D^0 \rightarrow K^\pm \pi^\mp$ decays. When a single uncertainty is listed, it corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been superseded and thus are not included in the HFLAV global fit (Section 70.7). The measurements with a dagger (\dagger) are not included in the HFLAV global fit due to much poorer precision.

Year	Experiment	$R (\times 10^{-3})$	$R_D (\times 10^{-3})$	$A_D (\%)$
2025	LHCb (8.4 fb $^{-1}$ D^* tag) [19]	—	3.427 ± 0.019	-0.66 ± 0.57
2025	LHCb (8.4 fb $^{-1}$ $B+D^*$ double-tagged) [20]	—	3.470 ± 0.051	0.9 ± 1.5
2014	Belle (976 fb $^{-1}$) [21]	3.86 ± 0.06	3.53 ± 0.13	—
2013	CDF (9.6 fb $^{-1}$) [22]	4.30 ± 0.05	3.51 ± 0.35	—
2007	BABAR (384 fb $^{-1}$) [23]	$3.53 \pm 0.08 \pm 0.04$	$3.03 \pm 0.16 \pm 0.10$	$-2.1 \pm 5.2 \pm 1.5$
HFLAV Fit Result [24]			3.436 ± 0.012	-0.81 ± 0.88
2018*	LHCb (5.0 fb $^{-1}$ D^* tag) [25]	—	3.454 ± 0.031	-0.01 ± 0.91
2017*	LHCb (3.0 fb $^{-1}$ $B+D^*$ double-tagged) [26]	—	3.48 ± 0.10	-3.15 ± 3.31
2013b*	LHCb (3.0 fb $^{-1}$ D^* tag) [27]	—	3.568 ± 0.066	-0.7 ± 1.9
2013a*	LHCb (1.0 fb $^{-1}$) [28]	4.25 ± 0.04	3.52 ± 0.15	—
2008*	CDF (1.5 fb $^{-1}$) [29]	4.15 ± 0.10	3.04 ± 0.55	—
2006*	Belle (400 fb $^{-1}$) [30]	$3.77 \pm 0.08 \pm 0.05$	3.64 ± 0.18	2.3 ± 4.7
2005 \dagger	FOCUS (234 evts) [31]	$4.29^{+0.63}_{-0.61} \pm 0.27$	$5.17^{+1.47}_{-1.58} \pm 0.76$	$13^{+33}_{-25} \pm 10$
2000 \dagger	CLEO (9.0 fb $^{-1}$) [32]	$3.32^{+0.63}_{-0.65} \pm 0.40$	$4.8 \pm 1.2 \pm 0.4$	$-1^{+16}_{-17} \pm 1$
1998 \dagger	E791 (5643 evts) [33]	$6.8^{+3.4}_{-3.3} \pm 0.7$	—	—

can potentially mix to \bar{D}^0 before decaying to $K^+ \pi^-$, and this introduces some dependence on the mixing parameters x and y . This dependence is seen explicitly in the observable

$$A_{K\pi}^{CP} \equiv \frac{|A(D_- \rightarrow K^- \pi^+)|^2 - |A(D_+ \rightarrow K^- \pi^+)|^2}{|A(D_- \rightarrow K^- \pi^+)|^2 + |A(D_+ \rightarrow K^- \pi^+)|^2}. \quad (70.38)$$

To lowest order in the mixing parameters [34],

$$A_{K\pi}^{CP} = \frac{2\sqrt{R_D} \cos \delta_{K\pi} + y}{1 + R}, \quad (70.39)$$

where R is defined in Eq. (70.34). Such measurements are discussed in Section 70.5.

70.3.1 Wrong-sign decays to multibody final states

For multibody final states, Eqs. (70.25)-(70.34) apply to each point in phase-space. Although x and y do not vary across phase-space, knowledge of the resonant substructure is needed to determine the strong phase difference δ at each point in order to extract x and y . Alternatively, experimental knowledge of the strong phase difference between A_f and \bar{A}_f or between $A_{\bar{f}}$ and $\bar{A}_{\bar{f}}$ in regions of phase space [35] allows one to determine x and y independent of the decay model of resonant substructure. This phase information can be measured in $e^+ e^- \rightarrow \psi(3770) \rightarrow \bar{D} D$ reactions in a manner similar to that for measuring $\delta_{K\pi}$, i.e., applying Eq. (70.37) to regions of phase space.

A time-dependent analysis of DCS $D^0 \rightarrow K^+ \pi^- \pi^0$ decays relative to CF $\bar{D}^0 \rightarrow K^+ \pi^- \pi^0$ decays by BABAR [36, 37] fit for both mixing parameters and the strong phase variation across the Dalitz

Table 70.3: Results for mixing parameters measured using $D^0 \rightarrow K^\pm \pi^\mp$ decays. When a single uncertainty is listed, it corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been superseded and thus are not included in the HFLAV global fit. The measurements with a dagger (\dagger) are not included in the HFLAV global fit due to much poorer precision.

Year	Experiment	No CP violation		Allowing for CP violation	
		$x'^2 (\times 10^{-3})$	$y' (\%)$	$c_{K\pi}, \Delta c_{K\pi}$ ($\times 10^{-4}$)	$c'_{K\pi}, \Delta c'_{K\pi}$ ($\times 10^{-6}$)
2025	LHCb (6, 8.4 fb $^{-1}$ D^* tag) [19]	—	—	52.8 ± 3.3 2.0 ± 3.4	12.0 ± 3.5 -0.7 ± 3.6
2025	LHCb (5.4, 8.4 fb $^{-1}$ $B+D^*$ double-tagged) [20]	0.00 ± 0.12	0.58 ± 0.16	$\left\{ \begin{array}{l} D^0: 0.08 \pm 0.15 \\ \bar{D}^0: -0.05 \pm 0.17 \end{array} \right.$	$\left\{ \begin{array}{l} 0.41 \pm 0.20 \\ 0.68 \pm 0.21 \end{array} \right.$
2014	Belle (976 fb $^{-1}$) [21]	0.09 ± 0.22	0.46 ± 0.34	—	—
2013	CDF (9.6 fb $^{-1}$) [22]	0.08 ± 0.18	0.43 ± 0.43	—	—
2007	BABAR (384 fb $^{-1}$) [23]	-0.22 ± 0.37	0.97 ± 0.54	$\left\{ \begin{array}{l} D^0: -0.24 \pm 0.52 \\ \bar{D}^0: -0.20 \pm 0.50 \end{array} \right.$	$\left\{ \begin{array}{l} 0.98 \pm 0.78 \\ 0.96 \pm 0.75 \end{array} \right.$
2006	Belle (400 fb $^{-1}$) [30]	$(0.18^{+0.21}_{-0.23})^*$	$(0.06^{+0.40}_{-0.39})^*$	< 0.72	$-2.8 < y' < 2.1$
2018*	LHCb (5.0 fb $^{-1}$ D^* tag) [25]	0.039 ± 0.027	0.528 ± 0.052	$\left\{ \begin{array}{l} D^0: 0.061 \pm 0.037 \\ \bar{D}^0: 0.016 \pm 0.039 \end{array} \right.$	$\left\{ \begin{array}{l} 0.501 \pm 0.074 \\ 0.554 \pm 0.074 \end{array} \right.$
2017*	LHCb (3.0 fb $^{-1}$ $B+D^*$ double tag) [26]	0.028 ± 0.310	0.46 ± 0.37	$\left\{ \begin{array}{l} D^0: -0.019 \pm 0.447 \\ \bar{D}^0: 0.079 \pm 0.433 \end{array} \right.$	$\left\{ \begin{array}{l} 0.581 \pm 0.526 \\ 0.332 \pm 0.523 \end{array} \right.$
2013b*	LHCb (3.0 fb $^{-1}$ D^* tag) [27]	0.055 ± 0.049	0.48 ± 0.10	$\left\{ \begin{array}{l} D^0: 0.049 \pm 0.070 \\ \bar{D}^0: 0.060 \pm 0.068 \end{array} \right.$	$\left\{ \begin{array}{l} 0.51 \pm 0.14 \\ 0.45 \pm 0.14 \end{array} \right.$
2013a*	LHCb (1.0 fb $^{-1}$) [28]	-0.09 ± 0.13	0.72 ± 0.24	—	—
2008*	CDF (1.5 fb $^{-1}$) [29]	-0.12 ± 0.35	0.85 ± 0.76	—	—
2005 \dagger	FOCUS (234 evts) [31]	< 8.3	$-7.2 < y' < 4.1$	< 8.0	$-11.2 < y' < 6.7$
2000 \dagger	CLEO (9.0 fb $^{-1}$) [32]	0.00 ± 0.23	$-2.3^{+1.3}_{-1.4}$	0.00 ± 0.23	$-2.5^{+1.4}_{-1.6}$
1998 \dagger	E791 (5643 evts) [33]	< 17	< 13	—	—

plot. They reported results $x'' = (2.61^{+0.57}_{-0.68} \pm 0.39)\%$ and $y'' = (-0.06^{+0.55}_{-0.64} \pm 0.34)\%$, where

$$x'' = x \cos \delta_{K\pi\pi^0} + y \sin \delta_{K\pi\pi^0} \quad (70.40)$$

$$y'' = y \cos \delta_{K\pi\pi^0} - x \sin \delta_{K\pi\pi^0}, \quad (70.41)$$

in analogy with x' , y' , and $\delta_{K\pi}$ for $D^0 \rightarrow K^+ \pi^-$ decays. Here, $\delta_{K\pi\pi^0}$ is the “reference” strong phase difference between amplitudes $A(\bar{D}^0 \rightarrow K^+ \rho^-)$ and $A(D^0 \rightarrow K^+ \rho^-)$, which cannot be determined in this analysis. However, $\delta_{K\pi\pi^0}$ can be measured using Eq. (70.37) and quantum-correlated measurements in $e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$ events of the branching fractions $B(D_+ \rightarrow K^+ \rho^-)$, $B(D_- \rightarrow K^+ \rho^-)$, $B(D^0 \rightarrow K^+ \rho^-)$, and $B(\bar{D}^0 \rightarrow K^+ \rho^-)$.

For decays $(D^0, \bar{D}^0) \rightarrow K^+ \pi^- \pi^+ \pi^-$, Belle measured $R = (0.324 \pm 0.008 \pm 0.007)\%$ [38]. A phase-space-integrated analysis from LHCb [39] measured the product of a “coherence factor” $R_D^{K3\pi}$ and the mixing parameter $y''_{K3\pi}$ with high statistics. This resulted in an observation of charm mixing

with a significance of 8.2σ .

Table 70.4: Results from time-dependent multibody analyses. The errors are statistical, systematic, and, when a third error is listed, due to the decay-model, respectively. The measurement with an asterisk (*) has been superseded and thus is not included in the HFLAV global fit. The measurement with a dagger (\dagger) is not included in the HFLAV global fit due to poorer precision. The 2019–2023 LHCb analyses fit for CP -violating parameters Δx and Δy ; the translation of these parameters to $|q/p|$ and ϕ is given in Ref. [40].

No CP Violation				
Year	Experiment	Final state(s)	x ($\times 10^{-3}$)	y ($\times 10^{-3}$)
2025	Belle II (951+408 fb $^{-1}$) [41]	$K_S^0 \pi^+ \pi^-$	$4.0 \pm 1.7 \pm 0.4$	$2.9 \pm 1.4 \pm 0.3$
2023	LHCb (5.4 fb $^{-1}$ B tag) [42]	$K_S^0 \pi^+ \pi^-$	$4.29 \pm 1.48 \pm 0.26$	$12.61 \pm 3.12 \pm 0.83$
2021	LHCb (5.4 fb $^{-1}$ D^* tag) [43]	$K_S^0 \pi^+ \pi^-$	$3.97 \pm 0.46 \pm 0.29$	$4.59 \pm 1.20 \pm 0.85$
2019	LHCb (3.0 fb $^{-1}$ B, D^* tags) [44]	$K_S^0 \pi^+ \pi^-$	$2.7 \pm 1.6 \pm 0.4$	$7.4 \pm 3.6 \pm 1.1$
2016	LHCb (1.0 fb $^{-1}$ D^* tag) [45]	$K_S^0 \pi^+ \pi^-$	$-8.6 \pm 5.3 \pm 1.7$	$0.3 \pm 4.6 \pm 1.3$
2016	BABAR (468 fb $^{-1}$) [46]	$\pi^+ \pi^- \pi^0$	$15 \pm 12 \pm 6$	$2 \pm 9 \pm 5$
2010	BABAR (469 fb $^{-1}$) [47]	$\left\{ \begin{array}{l} K_S^0 \pi^+ \pi^- \\ K_S^0 K^+ K^- \end{array} \right.$	$1.6 \pm 2.3 \pm 1.2 \pm 0.8$	$5.7 \pm 2.0 \pm 1.3 \pm 0.7$
2014*	Belle (921 fb $^{-1}$) [48]	$K_S^0 \pi^+ \pi^-$	$5.6 \pm 1.9^{+0.3+0.6}_{-0.9-0.9}$	$3.0 \pm 1.5^{+0.4+0.3}_{-0.5-0.6}$
2007*	Belle (540 fb $^{-1}$) [49]	$K_S^0 \pi^+ \pi^-$	$8.0 \pm 2.9^{+0.9+1.0}_{-0.7-1.4}$	$3.3 \pm 2.4^{+0.8+0.6}_{-1.2-0.8}$
2005 \dagger	CLEO (9.0 fb $^{-1}$) [50]	$K_S^0 \pi^+ \pi^-$	$19^{+32}_{-33} \pm 4 \pm 4$	$-14 \pm 24 \pm 8 \pm 4$
With CP Violation				
Year	Experiment	Final state(s)	$ q/p $	ϕ
2023	LHCb (5.4 fb $^{-1}$ B tag) [42]	$K_S^0 \pi^+ \pi^-$	$\left\{ \begin{array}{l} \Delta x \times 10^3 = \\ -0.77 \pm 0.93 \pm 0.28 \\ 0.996 \pm 0.052 \end{array} \right.$	$\left\{ \begin{array}{l} \Delta y \times 10^3 = \\ 3.01 \pm 1.92 \pm 0.26 \\ (3.2^{+2.7}_{-2.9})^\circ \end{array} \right.$
2021	LHCb (5.4 fb $^{-1}$ D^* tag) [43]	$K_S^0 \pi^+ \pi^-$	$\left\{ \begin{array}{l} \Delta x \times 10^3 = \\ -0.27 \pm 0.18 \pm 0.01 \\ 1.05^{+0.22}_{-0.17} \end{array} \right.$	$\left\{ \begin{array}{l} \Delta y \times 10^3 = \\ 0.20 \pm 0.36 \pm 0.13 \\ (-5.2^{+6.3}_{-9.2})^\circ \end{array} \right.$
2019	LHCb (3.0 fb $^{-1}$ B, D^* tags) [44]	$K_S^0 \pi^+ \pi^-$	$\left\{ \begin{array}{l} \Delta x \times 10^3 = \\ -0.53 \pm 0.70 \pm 0.22 \\ 0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05} \end{array} \right.$	$\left\{ \begin{array}{l} \Delta y \times 10^3 = \\ 0.6 \pm 1.6 \pm 0.3 \\ (-6 \pm 11 \pm 3^{+3}_{-4})^\circ \end{array} \right.$
2014	Belle (921 fb $^{-1}$) [48]	$K_S^0 \pi^+ \pi^-$	$0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}$	$(-6 \pm 11 \pm 3^{+3}_{-4})^\circ$
2007* \dagger	Belle (540 fb $^{-1}$) [49]	$K_S^0 \pi^+ \pi^-$	$0.86^{+0.30+0.06}_{-0.29-0.03} \pm 0.08$	$(-14^{+16+5+2}_{-18-3-4})^\circ$

\dagger This result allows for all types of CP violation and is superseded by Ref. [48], which assumes no direct CP violation in CF or DCS decays.

Both the sign and magnitude of x and y without strong phases entering can be determined by measuring the time-dependence of resonance structure for multibody D^0 decays to self-conjugate final states [49, 50]. This method has been used extensively for $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays, in which the DCS and CF amplitudes populate the same Dalitz plot. Assuming a decay model of intermediate resonances, this allows for fitting for strong phase differences and direct measurement of x and y . Alternatively, several LHCb $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ analyses [42–45] use values of strong phase differences measured in bins of phase space by CLEO-c [51] and BESIII [52, 53]; these (x, y) measurements are

independent of the decay model. The most recent LHCb analyses fit for CP -violating parameters Δx and Δy ; the translation of these parameters to $|q/p|$ and ϕ is given in Ref. [40]. Belle II [41] also used values of strong phase differences measured by BESIII [52, 53] to determine decay-model-independent values of x and y . Table 70.4 summarizes results from time-dependent analyses of self-conjugate multibody final states. World average values for x and y are given later, when discussing results from the HFLAV global fit.

BESIII recently updated their measurements of strong-phase differences for $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays using 7.9 fb^{-1} of data [54]; these are now the world's most precise values. BESIII has also measured strong-phase differences in bins of phase space for $(D^0, \bar{D}^0) \rightarrow K^+ K^- \pi^+ \pi^-$ decays [55] and $(D^0, \bar{D}^0) \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays [56]. Strong phase differences for the latter mode have also been measured using a small sample (0.82 fb^{-1}) of CLEO-c data [57]. These multibody self-conjugate final states can all be used to determine x and y .

70.4 Hadronic decays to CP eigenstates

When the final state f is a CP eigenstate, there is no distinction between f and \bar{f} . Thus, $A_f = A_{\bar{f}}$, $\bar{A}_{\bar{f}} = \bar{A}_f$, and $\lambda_f = \lambda_{\bar{f}}$. We denote final states with CP eigenvalues ± 1 by f_{\pm} . Decays to CP eigenstates are singly Cabibbo-suppressed and can proceed via both a tree amplitude and an internal-loop amplitude. These two amplitudes have different weak phases, and thus their interference leads to direct CP violation. As loop amplitudes are suppressed, direct CP violation is expected to be very small, and the presence of a weak phase is often neglected. For a negligible weak phase in the decay amplitude, $\lambda_{f_{\pm}} = \mp |q/p| e^{i\phi}$, where the choice of signs corresponds to $\phi = 0$ (rather than π) if CP is conserved.

The mixing parameter y can be measured by comparing the rate for D^0 decays to CP eigenstates such as $K^+ K^-$ with the rate to flavor eigenstates such as $K^- \pi^+$ [18]. If decays to $K^+ K^-$ have a shorter effective lifetime than those to $K^- \pi^+$, then $\Gamma_+ > \Gamma_-$. Since CP violation is very small, $\Gamma_2 > \Gamma_1$ and y is positive. Since mixing is very small, i.e., $x, y \ll 1$, the $(\Gamma t)^2$ terms in Eqs. (70.27) and (70.28) are often neglected. In this case the decay rates for $D^0 \rightarrow f_{\pm}$ and $\bar{D}^0 \rightarrow f_{\pm}$ are approximately exponential:

$$r_{\pm}(t) \propto \exp(-\Gamma_{\pm} t) \quad (70.42)$$

$$\bar{r}_{\pm}(t) \propto \exp(-\bar{\Gamma}_{\pm} t), \quad (70.43)$$

with effective decay widths

$$\Gamma_{\pm} = \Gamma \left(1 \pm \left| \frac{q}{p} \right| (y \cos \phi - x \sin \phi) \right) \quad (70.44)$$

$$\bar{\Gamma}_{\pm} = \Gamma \left(1 \pm \left| \frac{p}{q} \right| (y \cos \phi + x \sin \phi) \right). \quad (70.45)$$

The effective decay rate to a CP eigenstate for a data sample containing equal numbers of D^0 and \bar{D}^0 decays (e.g., an untagged sample with no production asymmetry) is

$$r_{\pm}(t) + \bar{r}_{\pm}(t) \propto e^{-(1 \pm y_{CP}) \Gamma t}, \quad (70.46)$$

where

$$y_{CP} = \frac{1}{2} \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) y \cos \phi - \frac{1}{2} \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) x \sin \phi \quad (70.47)$$

$$\approx y \cos \phi - A_M x \sin \phi. \quad (70.48)$$

Table 70.5: Results for y_{CP} and A_Γ from D^0 decays to CP eigenstates. When a single uncertainty is listed, it corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been superseded. The third row ($8.4 \text{ fb}^{-1} A_\Gamma$) lists the LHCb combination of the LHCb measurements listed below it.

Year	Experiment	Final state(s)	y_{CP} (%)	$A_\Gamma (\times 10^{-3})$
2024 [‡]	LHCb ($7.7 \text{ fb}^{-1} D^*$ tag) [59]	$\pi^+\pi^-\pi^0$	—	$0.13 \pm 0.63 \pm 0.24$
2022 [§]	LHCb ($6 \text{ fb}^{-1} D^*$ tag) [60]	$K^+K^- + \pi^+\pi^-$	$0.696 \pm 0.026 \pm 0.013$	—
2021	LHCb ($8.4 \text{ fb}^{-1} B, D^*$ tags) [61]	$K^+K^- + \pi^+\pi^-$	—	$0.10 \pm 0.11 \pm 0.03$
2021	LHCb ($6 \text{ fb}^{-1} D^*$ tag) [61]	$K^+K^- + \pi^+\pi^-$	—	$0.27 \pm 0.13 \pm 0.03$
2021	LHCb ($6 \text{ fb}^{-1} D^*$ tag) [61]	K^+K^-	—	$0.23 \pm 0.15 \pm 0.03$
2021	LHCb ($6 \text{ fb}^{-1} D^*$ tag) [61]	$\pi^+\pi^-$	—	$0.40 \pm 0.28 \pm 0.04$
2020	LHCb ($5.4 \text{ fb}^{-1} B$ tag) [62]	K^+K^-	—	$-0.43 \pm 0.36 \pm 0.05$
2020	LHCb ($5.4 \text{ fb}^{-1} B$ tag) [62]	$\pi^+\pi^-$	—	$0.22 \pm 0.70 \pm 0.08$
2020	Belle (976 fb^{-1}) [63]	$K_S^0 \omega$	$0.96 \pm 0.91^{+0.64}_{-0.62}$	—
2019	LHCb ($3 \text{ fb}^{-1} B$ tag) [64]	$K^+K^- + \pi^+\pi^-$	$0.57 \pm 0.13 \pm 0.09$	—
2017	LHCb ($3 \text{ fb}^{-1} D^*$ tag) [65]	$K^+K^- + \pi^+\pi^-$	—	$-0.13 \pm 0.28 \pm 0.10$
2017	LHCb ($3 \text{ fb}^{-1} D^*$ tag) [65]	K^+K^-	—	$-0.30 \pm 0.32 \pm 0.10$
2017	LHCb ($3 \text{ fb}^{-1} D^*$ tag) [65]	$\pi^+\pi^-$	—	$0.46 \pm 0.58 \pm 0.12$
2016	Belle (976 fb^{-1}) [66]	$K^+K^- + \pi^+\pi^-$	$1.11 \pm 0.22 \pm 0.09$	$-0.3 \pm 2.0 \pm 0.7$
2015	LHCb ($3 \text{ fb}^{-1} B$ tag) [67]	$K^+K^- + \pi^+\pi^-$	—	-1.25 ± 0.73
2015	LHCb ($3 \text{ fb}^{-1} B$ tag) [67]	K^+K^-	—	$-1.34 \pm 0.77^{+0.26}_{-0.34}$
2015	LHCb ($3 \text{ fb}^{-1} B$ tag) [67]	$\pi^+\pi^-$	—	$-0.92 \pm 1.45^{+0.25}_{-0.33}$
2015	BES III (2.9 fb^{-1}) [68]	$\left\{ \begin{array}{l} K^+K^-, \pi^+\pi^- \\ K_S^0 \pi^0, K_S^0 \pi^0 \pi^0 \\ K_S^0 \eta, K_S^0 \omega \end{array} \right.$	$-2.0 \pm 1.3 \pm 0.7$	—
2014	CDF (9.7 fb^{-1}) [69]	$K^+K^- + \pi^+\pi^-$	—	-1.2 ± 1.2
2014	CDF (9.7 fb^{-1}) [69]	K^+K^-	—	$-1.9 \pm 1.5 \pm 0.4$
2014	CDF (9.7 fb^{-1}) [69]	$\pi^+\pi^-$	—	$-0.1 \pm 1.8 \pm 0.3$
2012	BABAR (468 fb^{-1}) [70]	$K^+K^- + \pi^+\pi^-$	$0.72 \pm 0.18 \pm 0.12$	$0.9 \pm 2.6 \pm 0.6$
2009	Belle (673 fb^{-1}) [71]	$K_S^0 K^+K^-$	$0.11 \pm 0.61 \pm 0.52$	—
2002	CLEO (9.0 fb^{-1}) [72]	$K^+K^- + \pi^+\pi^-$	$-1.2 \pm 2.5 \pm 1.4$	—
2000	FOCUS (1×10^6 evts) [73]	K^+K^-	$3.42 \pm 1.39 \pm 0.74$	—
1999	E791 (2×10^{10} evts) [74]	K^+K^-	$0.73 \pm 2.89 \pm 1.03$	—
HFLAV Average [75]			0.697 ± 0.028	0.089 ± 0.113
2013*	LHCb ($1.0 \text{ fb}^{-1} D^*$ tag) [76]	K^+K^-	—	$-0.35 \pm 0.62 \pm 0.12$
2013*	LHCb ($1.0 \text{ fb}^{-1} D^*$ tag) [76]	$\pi^+\pi^-$	—	$0.33 \pm 1.06 \pm 0.14$
2011* [¶]	LHCb ($29 \text{ pb}^{-1} D^*$ tag) [77]	K^+K^-	$0.55 \pm 0.63 \pm 0.41$	$-5.9 \pm 5.9 \pm 2.1$
2009*	BABAR (384 fb^{-1}) [78]	K^+K^-	$1.16 \pm 0.22 \pm 0.18$	—
2008*	BABAR (384 fb^{-1}) [79]	$K^+K^- + \pi^+\pi^-$	$1.03 \pm 0.33 \pm 0.19$	$2.6 \pm 3.6 \pm 0.8$
2007*	Belle (540 fb^{-1}) [80]	$K^+K^- + \pi^+\pi^-$	$1.31 \pm 0.32 \pm 0.25$	$0.1 \pm 3.0 \pm 1.5$
2003*	BABAR (91 fb^{-1}) [81]	$K^+K^- + \pi^+\pi^-$	$0.8 \pm 0.4^{+0.5}_{-0.4}$	—
2001*	Belle (23.4 fb^{-1}) [82]	K^+K^-	$-0.5 \pm 1.0^{+0.7}_{-0.8}$	—

[‡]This measurement uses the $D^0 \rightarrow \pi^+\pi^-\pi^0$ CP -even fraction measured in Ref. [58].

[§]This measurement has sufficient precision that y_{CP} for the normalization channel $D^0 \rightarrow K^-\pi^+$ must be accounted for. HFLAV accounts for this small correction in their global fit.

[¶]This result for y_{CP} is not superseded, but it is not included in the HFLAV average due to having some correlations with the result of Ref. [64] but much worse precision.

If CP is conserved ($\phi = 0$), $y_{CP} = y$. Equation (70.46) implies that y_{CP} can be determined from the decay-time dependence of either CP -even or CP -odd decays:

$$y_{CP} = \frac{\Gamma_+}{\Gamma} - 1 = 1 - \frac{\Gamma_-}{\Gamma}. \quad (70.49)$$

Most y_{CP} measurements have used $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays, which are CP -even, measured relative to $D^0 \rightarrow K^-\pi^+$. Belle measured y_{CP} also using $D^0 \rightarrow K_S^0 \omega$ decays [63], which are CP -odd, and $D^0 \rightarrow K_S^0 K^+K^-$ decays [71], which are dominated by the CP -odd final state $K_S^0 \phi$. Table 70.5 summarizes the status of measurements. The most recent y_{CP} measurement from LHCb has sufficient precision that the tiny effect of mixing upon $D^0(t) \rightarrow K^-\pi^+$ decays, which are used in the measurement to determine Γ , must be accounted for. The measurement directly yields

$$\frac{\Gamma_+}{\Gamma_{K^-\pi^+}} - 1 = \frac{1 + y_{CP}}{1 + y_{CP}^{K\pi}} - 1 \approx y_{CP} - y_{CP}^{K\pi}, \quad (70.50)$$

where the correction $y_{CP}^{K\pi}$ is $\sim 10^{-4}$. This correction was first pointed out in Ref. [83]; it is calculated in terms of parameters ($x, y, |q/p|, \phi$) in Ref. [84].

In addition to y_{CP} , the asymmetry in decay rates to CP -even final states (A_Γ) has been measured [61, 66, 69, 70, 77]:

$$A_\Gamma \equiv \frac{\Gamma_+ - \bar{\Gamma}_+}{\Gamma_+ + \bar{\Gamma}_+} = \frac{(1/\tau_+) - (1/\bar{\tau}_+)}{(1/\tau_+) + (1/\bar{\tau}_+)} = \frac{\bar{\tau}_+ - \tau_+}{\bar{\tau}_+ + \tau_+} \quad (70.51)$$

$$\approx \frac{1}{2} \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) y \cos \phi - \frac{1}{2} \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) x \sin \phi \quad (70.52)$$

$$\approx A_M y \cos \phi - x \sin \phi. \quad (70.53)$$

If CP is conserved ($A_M = 0, \phi = 0$), then $A_\Gamma = 0$.

There is a contribution to Eq. (70.53) from direct CP violation, i.e., $|\bar{A}_f/A_f| \neq 1$ [1, 85]. For $f = K^+K^-$ and $\pi^+\pi^-$, this contribution can be estimated from measurements of $A_{CP}(K^+K^-)$ and $A_{CP}(\pi^+\pi^-)$ (see below) and is much smaller than the current uncertainty on A_M ; thus we neglect it here. We note that, when averaging A_Γ measurements over K^+K^- and $\pi^+\pi^-$ final states, the contribution from direct CP violation nominally cancels, as it is expected to have the same magnitude for K^+K^- and $\pi^+\pi^-$ but opposite signs due to U -spin symmetry [1].

The observable A_Γ is an asymmetry in the full decay widths. An asymmetry in *partial* widths is referred to as A_{CP} and depends on the final state:

$$A_{CP} \equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow \bar{f})}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow \bar{f})}. \quad (70.54)$$

Unlike A_Γ , which is measured by fitting decay-time distributions, A_{CP} is measured by fitting for signal yields and, aside from acceptance effects, does not require measuring decay times. For neutral D decays, A_{CP} receives contributions from both direct and indirect CP violation: $A_{CP}(D^0 \rightarrow f) = A_{CP}^f + A_{CP}^{\text{indirect}}$. The indirect contribution depends on x and y :

$$A_{CP}^{\text{indirect}} = \frac{1}{2} \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) x \sin \phi - \frac{1}{2} \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) y \cos \phi = -A_\Gamma. \quad (70.55)$$

Numerous measurements of A_{CP} for decays to CP eigenstates are listed in this *Review* [86]. Table 70.6 summarizes the status of measurements of the *difference* in A_{CP} for $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays: $\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$. U -spin symmetry predicts

Table 70.6: Results for A_{CP} and the difference ΔA_{CP} between $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays. When a single uncertainty is listed, it corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been either superseded or combined with subsequent results and thus are not included in the HFLAV global fit.

Year	Experiment	$A_{CP} (\times 10^{-3})$
2024	LHCb $D^0 \rightarrow K^+K^-$ (5.7 fb $^{-1}$) [89]	0.68 ± 0.56
		$\Delta A_{CP} (\times 10^{-3})$
2019	LHCb (8.9 fb $^{-1}$ B, D^* tags) [88]	-1.54 ± 0.29
2013	CDF (9.7 fb $^{-1}$ D^* tag) [90]	$-6.2 \pm 2.1 \pm 1.0$
2008	BABAR (386 fb $^{-1}$) [91]	$2.4 \pm 6.2 \pm 2.6$
2008	Belle (540 fb $^{-1}$) [92]	$-8.6 \pm 6.0 \pm 0.7$
2016*	LHCb (3.0 fb $^{-1}$ D^* tag) [93]	$-1.0 \pm 0.8 \pm 0.3$
2014*	LHCb (3.0 fb $^{-1}$ B tag) [94]	$1.4 \pm 1.6 \pm 0.8$
2013*	LHCb (1.0 fb $^{-1}$ B tag) [95]	$4.9 \pm 3.0 \pm 1.4$
2012*	LHCb (0.62 fb $^{-1}$ D^* tag) [96]	$-8.2 \pm 2.1 \pm 1.1$
2012 ‡	Belle (976 fb $^{-1}$) [97]	$-8.7 \pm 4.1 \pm 0.6$

‡ This preliminary result was not published and thus is not included in the HFLAV global fit.

$A_{CP}(KK) = -A_{CP}(\pi\pi)$ [87]; consequently, even accounting for U -spin-breaking, ΔA_{CP} should increase any direct CP asymmetry present. The difference is also advantageous experimentally, as several systematic uncertainties cancel. As A_{CP}^{indirect} is independent of the final state, it subtracts out of ΔA_{CP} . However, at hadron experiments such as LHCb, there is a difference in efficiencies between K^+K^- and $\pi^+\pi^-$ such that $\langle t \rangle_{KK} \neq \langle t \rangle_{\pi\pi}$, i.e., the mean decay times differ slightly. This difference introduces a small contribution to ΔA_{CP} from A_{CP}^{indirect} [85]. A measurement of ΔA_{CP} from LHCb [88] based on 8.9 fb $^{-1}$ of data differs from zero with a statistical significance of 5.3σ ; this measurement constitutes the first (and only) observation of CP violation in charm decays. A subsequent measurement by LHCb of $A_{CP}(K^+K^-)$ [89] yielded $(0.068 \pm 0.056)\%$, which is consistent with zero. Taken together, these results suggest that the large value of ΔA_{CP} is due to $D^0 \rightarrow \pi^+\pi^-$ decays, and that the expectation $A_{CP}(KK) \approx -A_{CP}(\pi\pi)$ is violated. However, the uncertainty on $A_{CP}(K^+K^-)$ is sufficiently large that the central value can be explained as a statistical fluctuation at the level of $\sim 2.3\sigma$. These CP asymmetries are included in HFLAV's global fit for charm mixing parameters, as discussed below.

70.5 Quantum-correlated $D^0\bar{D}^0$ Analyses

Measurements of R_D , $\cos \delta_{K\pi}$, $\sin \delta_{K\pi}$, x , and y can be obtained from a combined fit to time-integrated yields of single-tagged (ST) and double-tagged (DT) $D^0\bar{D}^0$ events produced at the $\psi(3770)$ resonance. Single-tagged events are those in which either the D^0 or \bar{D}^0 decay is reconstructed (identified), and the other neutral D decays generically. Double-tagged events are those in which both the D^0 and \bar{D}^0 decays are identified. Due to quantum correlations, the decay of a D^0 , \bar{D}^0 , D_+ , or D_- projects the other neutral D into a state \bar{D}^0 , D^0 , D_- , or D_+ , respectively. Neglecting CP violation, states D_- and D_+ are 50% D^0 and 50% \bar{D}^0 ; thus the D_\pm decay rates include an interference term proportional to $\sqrt{R_D} \cos \delta_{K\pi}$. The interference also includes contributions from $D^0\text{-}\bar{D}^0$ mixing and thus provides sensitivity to x and y . An example is the parameter $A_{K\pi}^{CP}$ in Eq. (70.39). For details of this method, see Refs. [4–8].

BESIII has reported results using 2.9 fb $^{-1}$ of $e^+e^- \rightarrow \psi(3770)$ data, where the quantum-

correlated $D^0\bar{D}^0$ pairs are produced in a $C=-1$ state. They measure $y_{CP} = (-2.0 \pm 1.3 \pm 0.7)\%$ [68] from DT yields using a CP -eigenstate tag for one D and a flavor-specific semileptonic tag for the other; and they measure $A_{K\pi}^{CP} = (13.2 \pm 1.1 \pm 0.7)\%$ [98] from DT yields using a CP tag for one D and a $K^\pm\pi^\mp$ tag for the other. For y_{CP} , the CP eigenstates used are K^-K^+ (f_+), $\pi^+\pi^-$ (f_+), $K_S^0\pi^0\pi^0$ (f_+), $K_S^0\pi^0$ (f_-), $K_S^0\eta$ (f_-), and $K_S^0\omega$ (f_-). For $A_{K\pi}^{CP}$, seven additional CP eigenstates are included: $\pi^0\pi^0$ (f_+), $K_S^0\eta'$ (f_-), $K_S^0\phi$ (f_-), $K_L^0\pi^0$ (f_+), $K_L^0\omega$ (f_+), $K_L^0\pi^0\pi^0$ (f_-), and $\pi^+\pi^-\pi^0$ (mixed CP). Using Eq. (70.39) and external inputs for the CP -even fraction of $D^0 \rightarrow \pi^+\pi^-\pi^0$ (from Ref. [58]) and values of R_D and y (from HFLAV [13]), BESIII obtains $\delta_{K\pi} = (7.6^{+10.4}_{-11.6})^\circ$ [98].

Recently, BESIII reported results (unpublished) using 7.13 fb^{-1} of data collected at center-of-mass energies in the range 4.13–4.12 GeV [99]. This range allows for the production of $D\bar{D}$, $D^*\bar{D}$, \bar{D}^*D , and $D^*\bar{D}^*$ pairs, and the resulting $D^0\bar{D}^0$ pair can be in either a CP -even or CP -odd state. The strong phase difference $\delta_{K\pi}$ is determined using $D^0\bar{D}^0$ pairs decaying to $K^-\pi^+$ versus K^+K^- (f_+), $\pi^+\pi^-$ (f_+), $K^0\pi^0$ (f_-), $\pi^+\pi^-\pi^0$ (nearly f_+), and $K_S^0\pi^+\pi^-$ (mixed CP) final states. Using external inputs for R_D and y , the analysis obtains similar precision for $\delta_{K\pi}$ as that of Ref. [98], with a higher central value of 12.8° .

CLEO-c has reported results using 0.82 fb^{-1} of $e^+e^- \rightarrow \psi(3770)$ data [100–102]. The values for y , R_M , $\cos\delta_{K\pi}$, and $\sin\delta_{K\pi}$ are obtained from a combined fit to the ST (hadronic only) and DT yields. The DT yields include events in which one D is reconstructed in a hadronic mode and the other D is partially reconstructed in flavor-specific $D \rightarrow K^\mp e^\pm\nu$ and $D \rightarrow K^\mp\mu^\pm\nu$ modes. The CLEO-c analysis obtains $\cos\delta_{K\pi} = 0.81^{+0.22}_{-0.18}{}^{+0.07}_{-0.05}$ and $\sin\delta_{K\pi} = -0.01 \pm 0.41 \pm 0.04$. These fits allow $\cos\delta_{K\pi}$ and $\sin\delta_{K\pi}$ (and also x^2) to be unphysical. Constraining $\cos\delta_{K\pi}$ and $\sin\delta_{K\pi}$ to the physical range $[-1, +1]$ (i.e., interpreting $\delta_{K\pi}$ as an angle) and also using external inputs for x , y , and y_{CP} from HFLAV 2012 [103], CLEO-c obtains $\delta_{K\pi} = (18^{+11}_{-17})^\circ$ [102].

70.6 Summary of Experimental Results

The first evidence for D^0 - \bar{D}^0 mixing was obtained in 2007 by Belle [80] and BABAR [23]. These results were confirmed by CDF [104] and, much later, by LHCb [28]. There are now numerous measurements of D^0 - \bar{D}^0 mixing with various levels of precision. Using $D^0 \rightarrow K^+\pi^-$ decays, CDF [22], LHCb [27, 28], and Belle [21] each exclude the no-mixing hypothesis by more than five standard deviations. The most significant observation of mixing (8.2σ significance) was made by LHCb using $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$ decays [39]. However, strong phase differences for this multibody decay are not known, and thus x and y cannot be extracted. The most precise measurement of y is essentially LHCb's measurement of y_{CP} using $D^0 \rightarrow (K^+K^-, \pi^+\pi^-)$ decays [60], and the most precise measurement of x is LHCb's measurement using $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays [43]. This measurement resulted in the first direct observation ($> 5\sigma$ significance) of dispersive mixing ($x \neq 0$).

These experimental measurements establish that D^0 and \bar{D}^0 mesons mix. In the Standard Model, this mixing is dominated by long-distance amplitudes, which are difficult to calculate. If one assumes the observed mixing is due to non-Standard Model processes, significant constraints on new physics models can be obtained [105]. A significant limitation to interpreting charm mixing in terms of new physics is the theoretical uncertainty on Standard Model predictions [106, 107]. We note that the HFLAV global fit results for x and y (see below) indicate that charm mixing is at the upper end of the range of predictions. The current situation would benefit from knowledge of strong phase differences between $\bar{D}^0 \rightarrow K^+\pi^-\pi^0$ and $D^0 \rightarrow K^+\pi^-\pi^0$ decays, and between $\bar{D}^0 \rightarrow K^+\pi^-\pi^+\pi^-$ and $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$ decays. Such knowledge would allow one to extract x and y from measurements of $(x''_{K2\pi}, y''_{K2\pi})$ and $(x''_{K3\pi}, y''_{K3\pi})$ performed for these modes.

With regard to CP violation, by combining separate measurements from two data sets totalling 8.9 fb^{-1} of data, LHCb observed CP violation in D decays for the first time [88]. The observable measured, $\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$, is dominated by direct CP violation; contribu-

tions from indirect CP violation mostly cancel. The amount of direct CP violation measured is small: $\Delta A_{CP} = (-0.154 \pm 0.029)\%$. Several theory calculations [108–110] indicate that this value is consistent with the Standard Model, although new physics contributions cannot be excluded (see e.g., [111, 112]). A subsequent LHCb measurement of $A_{CP}(K^+K^-)$ differs from zero by only 1.2σ [89]; thus, the ΔA_{CP} value is interpreted as indicating direct CP violation in $D^0 \rightarrow \pi^+\pi^-$ decays. Among all measurements, there is no evidence so far of indirect CP violation in the D^0 - \bar{D}^0 system.

70.7 HFLAV Global Fit for Charm Mixing Parameters

The Heavy Flavor Averaging Group (HFLAV) performs global fits to all relevant mixing measurements to obtain world average values for the following: mixing parameters x and y or alternatively x_{12} and y_{12} ; strong phase differences $\delta_{K\pi}$ and $\delta_{K\pi\pi^0}$; the ratio R_D of $\Gamma(D^0 \rightarrow K^+\pi^-)$ and $\Gamma(\bar{D}^0 \rightarrow K^+\pi^-)$ partial widths; direct CP -violating parameters $A_D(K^+\pi^-)$, $A_{CP}(D^0 \rightarrow K^+K^-) \equiv A_K$, and $A_{CP}(D^0 \rightarrow \pi^+\pi^-) \equiv A_\pi$; and indirect CP -violating parameters $|q/p|$ and ϕ or alternatively ϕ^M , ϕ^I , and their difference $\phi_{12} \equiv \phi^M - \phi^I$. Three separate fits are performed: (a) assuming no indirect CP violation; (b) assuming no subleading amplitudes in indirect CP violation (the “superweak” model); (c) allowing for all CP violation. Details of these fits are given in Ref. [13]. For fit (b) in which there are no subleading amplitudes, one can derive the relation $\tan \phi = (x/y) \cdot (1 - |q/p|^2)/(1 + |q/p|^2)$ [2, 3, 113]; this reduces four fitted parameters to three. Alternatively, one can fit for the three parameters x_{12} , y_{12} , and ϕ_{12} . However, the direct CP violation observed in ΔA_{CP} implies that subleading amplitudes play a role in Γ_{12} , the off-diagonal element of the decay matrix.

The fits use Belle, BABAR, CDF, and LHCb measurements of $D^0 \rightarrow K^{(*)+}\ell^-\bar{\nu}$, K^+K^- , $\pi^+\pi^-$, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, $K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$, and $\pi^+\pi^-\pi^0$ decays, as well as CLEO-c and BESIII measurements of $\cos \delta$, $\sin \delta$, and $A_{CP}(K^+\pi^-)$ obtained from quantum-correlated measurements of branching fractions in $e^+e^- \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0$ reactions. Correlations among observables are taken into account by using the error matrices provided by the experiments. Three observables input to the fit are themselves world average values calculated by HFLAV: $R_M = (x^2 + y^2)/2$ from $D^0 \rightarrow K^{(*)+}\ell^-\bar{\nu}$ decays (Table 70.1), and y_{CP} and A_I from $D^0 \rightarrow f_{CP}$ decays (Table 70.5). A measurement of R_M by LHCb using $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$ decays is input separately.

The results of the fit as of September, 2025 are listed in Table 70.7. Confidence contours in the two dimensions (x, y) and $(|q/p|, \phi)$ resulting from the fit are plotted in Fig. 70.1. These contours are obtained by allowing, for any point in the two-dimensional plane, all other floated parameters to take their preferred values. The 1σ – 5σ boundaries drawn are the loci of points in which the χ^2 rises above the minimum by 2.30, 6.18, 11.83, 19.33, and 28.67 units. The fit excludes the no-mixing point $x = y = 0$ at more than 11.5σ . The fit is consistent with CP conservation ($|q/p| = 1$, $\phi = 0$) at the 2σ level. The χ^2 of the all- CP -violation-allowed fit is 70.7 for $59 - 10 = 49$ degrees of freedom, which is considered satisfactory. One-dimensional likelihood functions for parameters are obtained by allowing, for any value of the parameter, all other floated parameters to take their preferred values. The resulting likelihood functions give the 95% C.L. intervals listed in Table 70.7.

From the results of the HFLAV fit, one concludes the following: (1) since CP violation is small and y_{CP} is positive, the CP -even state is shorter-lived, as in the $K^0\bar{K}^0$ system; but since x is positive, the CP -even state is heavier, which is opposite to the $K^0\bar{K}^0$ system. (2) The strong phase difference $\delta_{K\pi}$ is nonzero: the 95% C.L. interval is $8.1^\circ < \delta_{K\pi} < 20.9^\circ$, with a preferred value of 14.8° . (3) While direct CP violation has been observed ($\Delta A_{CP} \neq 0$), there is still no evidence for indirect CP violation, i.e., $|q/p| \neq 1$ or $\phi \neq 0$ or $\phi^M \neq 0$ or $\phi^I \neq 0$. Observing such CP violation at the current level of experimental sensitivity would hint at new physics.

Finally, we note that the global fit to charm mixing measurements can be expanded to include

Table 70.7: HFLAV global fit results (see text and Ref. [24]). The upper two sections list mixing and indirect CP violation (CPV) parameters; the third section lists strong phases and the ratio of decay rates R_D ; and the bottom section lists direct CP violation parameters.

Parameter	No indirect CPV	No subleading amplitudes in indirect CPV	All CPV allowed	95% C.L. Interval (all CPV)
	Fit (a)	Fit (b)	Fit (c)	
x (%)	0.398 ± 0.044	0.405 ± 0.043	0.405 ± 0.043	[0.320, 0.489]
y (%)	0.636 ± 0.024	0.638 ± 0.023	0.636 ± 0.024	[0.590, 0.682]
$ q/p $	1	1.002 ± 0.005	0.983 ± 0.015	[0.96, 1.01]
ϕ ($^\circ$)	0	-0.08 ± 0.19	-1.51 ± 1.04	[-3.63, 0.51]
x_{12} (%)	0.398 ± 0.044	0.405 ± 0.043	0.406 ± 0.043	[0.321, 0.490]
y_{12} (%)	0.636 ± 0.024	0.638 ± 0.023	0.635 ± 0.024	[0.589, 0.682]
ϕ^M ($^\circ$)	0	0.27 ± 0.66	-0.03 ± 0.70	[-1.48, 1.35]
ϕ^{Γ} ($^\circ$)	0	0	2.03 ± 1.53	[-0.94, 5.08]
$\delta_{K\pi}$ ($^\circ$)	14.9 ± 3.2	15.2 ± 3.2	14.8 ± 3.2	[8.1, 20.9]
$\delta_{K\pi\pi}$ ($^\circ$)	24.7 ± 22.2	24.4 ± 22.0	24.2 ± 22.1	[-20.6, 65.5]
R_D (%)	0.344 ± 0.001	0.344 ± 0.001	0.344 ± 0.001	[0.341, 0.346]
A_D (%)	0	0	-0.81 ± 0.88	[-2.54, 0.92]
A_π (%)	0.212 ± 0.051	0.222 ± 0.057	0.225 ± 0.057	[0.11, 0.34]
A_K (%)	0.055 ± 0.043	0.065 ± 0.051	0.068 ± 0.051	[-0.03, 0.17]
$\chi^2/\text{d.o.f.}$	$72.817/(59-7)$ = 1.40	$72.797/(59-8)$ = 1.43	$70.711/(59-10)$ = 1.44	

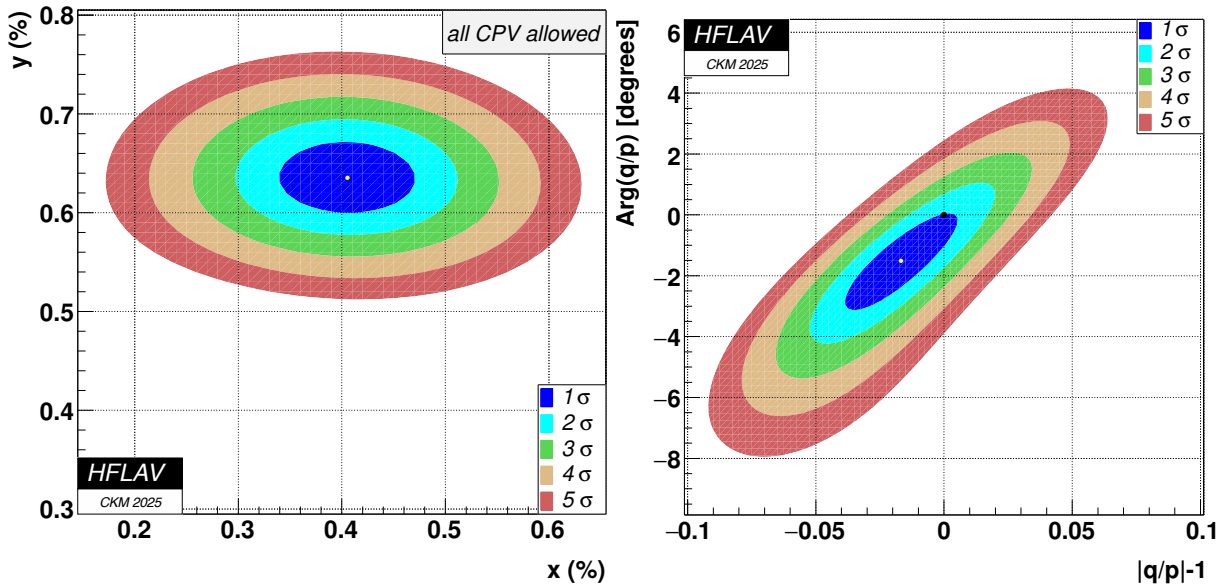


Figure 70.1: Two-dimensional 1σ - 5σ contours for (x, y) (left) and for $(|q/p|, \phi)$ (right) from the HFLAV global fit [24]. The black dot in the right plot denotes the no- CP -violation point.

$B \rightarrow (D^0, \bar{D}^0)K$ and $B \rightarrow (D^0, \bar{D}^0)\pi$ measurements; this larger fit is performed to determine the CKM angle ϕ_3 (or γ) [114, 115]. When $D^0 \rightarrow K^\pm\pi^\mp$, the B decay data is sensitive to the strong phase $\delta_{K\pi}$, and the fit gives improved precision for $\delta_{K\pi}$ [115]. This improvement leads to a

small improvement in the precision for y (from measurements of y' via $D^0 \rightarrow K^+\pi^-$ decays and Eq. (70.32)).

70.8 Future Data

Current mixing and CP violation results are based mainly upon the following:

- CLEO-c (0.82 fb^{-1}) and BESIII (2.9 fb^{-1}) data recorded in $e^+e^- \rightarrow \psi(3770)$ reactions;
- BABAR (384 fb^{-1}), Belle (976 fb^{-1}), and Belle II (408 fb^{-1}) data recorded in $e^+e^- \rightarrow \Upsilon(4S)$ reactions;
- CDF (9.6 fb^{-1}) data of $\bar{p}p$ collisions at $\sqrt{s} = 2 \text{ TeV}$; and
- LHCb Run 1 (3.0 fb^{-1} , $\sqrt{s} = 7, 8 \text{ TeV}$) and Run 2 (6 fb^{-1} , $\sqrt{s} = 13 \text{ TeV}$) data of pp collisions.

BESIII has recently completed its goal of accumulating 20 fb^{-1} of $e^+e^- \rightarrow \psi(3770)$ data. These data should provide numerous strong phase measurements that would enable additional model-independent determinations of mixing parameters from Belle II and LHCb. In 2019 Belle II began a long-range program to accumulate 50 ab^{-1} of $e^+e^- \rightarrow \Upsilon(4S)$ data [116]. To date, the experiment has recorded about 0.6 ab^{-1} of data and is expected to record several ab^{-1} over the next few years. At LHCb, Run 2 was completed in 2018, Run 3 is now in progress, and Run 4 is planned for 2030-33 [117]. The goal for Runs 3+4 is to accumulate an additional 50 fb^{-1} of pp data at $\sqrt{s} \approx 14 \text{ TeV}$ [118]. These data, along with the substantial e^+e^- dataset from Belle II, should provide more precise measurements of D^0 - \bar{D}^0 mixing and direct CP violation, and hopefully uncover indirect CP violation in the D^0 - \bar{D}^0 system.

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