

## 84. Extra Dimensions

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### 84.1 Introduction

Proposals for a spacetime with more than three spatial dimensions date back to the 1920s, mainly through the work of Kaluza and Klein, in an attempt to unify the forces of nature [1]. Although their initial idea failed, the formalism that they and others developed is still useful nowadays. Around 1980, string theory proposed again to enlarge the number of space dimensions, this time as a requirement for describing a consistent theory of quantum gravity. The extra dimensions were supposed to be compactified at a scale close to the Planck scale, and thus not testable experimentally in the near future.

A different approach was given by Arkani-Hamed, Dimopoulos, and Dvali (ADD) in their seminal paper in 1998 [2], where they showed that the weakness of gravity could be explained by postulating two or more flat extra dimensions in which only gravity could propagate. The size of these extra dimensions should range between roughly a millimeter and  $\sim 1/\text{TeV}$ , leading to possible observable consequences in current and future experiments. A year later, Randall and Sundrum (RS) [3] found a new possibility using a warped geometry, postulating a five-dimensional Anti-de Sitter (AdS) spacetime with a compactification scale of order  $1/\text{TeV}$ . The origin of the smallness of the electroweak scale versus the Planck scale was explained by the gravitational redshift factor present in the warped AdS metric. As in the ADD model, originally only gravity was assumed to propagate in the extra dimensions, although it was soon clear that this was not necessary in warped extra dimensions and also the SM gauge fields [4, 5] and SM fermions [6, 7] could propagate in the five-dimensional spacetime.

The physics of warped extra-dimensional models has an alternative interpretation by means of the AdS/Conformal Field Theory (CFT) correspondence [8–10]. Models with warped extra dimensions are related to four-dimensional strongly-interacting theories, allowing an understanding of the properties of five-dimensional fields as those of four-dimensional composite states [11]. This approach has opened new directions for tackling outstanding questions in particle physics, such as the flavor problem, grand unification, and the origin of electroweak symmetry breaking or supersymmetry breaking.

#### 84.1.1 *Experimental Constraints*

Constraints on extra-dimensional models arise from astrophysical and cosmological considerations, tabletop experiments exploring gravity at sub-mm distances, and collider experiments. Collider limits on extra-dimensional models are dominated by LHC results, which can be found on the public WWW pages of ATLAS [12] and CMS [13]. This review includes the most recent limits, most of which are published results based on  $140 \text{ fb}^{-1}$  LHC data collected in 2015–18 at a center-of-mass energy of 13 TeV and legacy results from  $20 \text{ fb}^{-1}$  of 8 TeV data collected in Run 1. For most of the models, Run 2 results surpass the sensitivity of Run 1.

#### 84.1.2 *Kaluza-Klein Theories*

Field theories with compact extra dimensions can be written as theories in ordinary four dimensions (4D) by performing a Kaluza-Klein (KK) reduction. As an illustration, consider a simple example, namely a field theory of a complex scalar in flat five-dimensional (5D) spacetime. The

action will be given by <sup>1</sup>

$$S_5 = - \int d^4x dy M_5 \left[ |\partial_\mu \phi|^2 + |\partial_y \phi|^2 + \lambda_5 |\phi|^4 \right], \quad (84.1)$$

where  $y$  refers to the extra (fifth) dimension. A universal scale  $M_5$  has been extracted in front of the action in order to keep the 5D field with the same mass-dimension as in 4D. This theory is perturbative for energies  $E \lesssim \ell_5 M_5 / \lambda_5$  where  $\ell_5 = 24\pi^3$  [14].

Let us now consider that the fifth dimension is compact with the topology of a circle  $S^1$  of radius  $R$ , which corresponds to the identification of  $y$  with  $y + 2\pi R$ . In such a case, the 5D complex scalar field can be expanded in a Fourier series:

$$\phi(x, y) = \frac{1}{\sqrt{2\pi R M_5}} \sum_{n=-\infty}^{\infty} e^{iny/R} \phi^{(n)}(x), \quad (84.2)$$

that, inserted in Eq. (84.1) and integrating over  $y$ , gives

$$S_5 = S_4^{(0)} + S_4^{(n)}, \quad (84.3)$$

where

$$S_4^{(0)} = - \int d^4x \left[ |\partial_\mu \phi^{(0)}|^2 + \lambda_4 |\phi^{(0)}|^4 \right], \quad (84.4)$$

and,

$$S_4^{(n)} = - \int d^4x \sum_{n \neq 0} \left[ |\partial_\mu \phi^{(n)}|^2 + \left(\frac{n}{R}\right)^2 |\phi^{(n)}|^2 \right] \\ + \text{quartic interactions}. \quad (84.5)$$

The  $n = 0$  mode self-coupling is given by

$$\lambda_4 = \frac{\lambda_5}{2\pi R M_5}. \quad (84.6)$$

The above action corresponds to a 4D theory with a massless scalar  $\phi^{(0)}$ , referred to as the zero mode, and an infinite tower of massive modes  $\phi^{(n)}$  with  $n > 0$ , known as KK modes. The KK reduction thus allows a treatment of 5D theories as 4D field theories with an infinite number of fields. At energies smaller than  $1/R$ , the KK modes can be neglected, leaving the zero-mode action of Eq. (84.4). The strength of the interaction of the zero-mode, given by Eq. (84.6), decreases as  $R$  increases. Thus, for a large extra dimension  $R \gg 1/M_5$ , the massless scalar is very weakly coupled.

## 84.2 Large Extra Dimensions for Gravity

### 84.2.1 The ADD Scenario

The ADD scenario [2, 15] (for a review see, for example, [16]) assumes a  $D = 4 + \delta$  dimensional spacetime, with  $\delta$  compactified spatial dimensions. The apparent weakness of gravity arises since it propagates in the higher-dimensional space. The SM is assumed to be localized in a 4D subspace, a 3-brane, as can be found in certain string theory constructions [17, 18]. Gravity is described by the Einstein-Hilbert action in  $D = 4 + \delta$  spacetime dimensions

$$S_D = - \frac{\bar{M}_D^{2+\delta}}{2} \int d^4x d^\delta y \sqrt{-g} \mathcal{R} + \int d^4x \sqrt{-g_{\text{ind}}} \mathcal{L}_{\text{SM}}, \quad (84.7)$$

<sup>1</sup>Our convention for the metric is  $\eta_{MN} = \text{Diag}(-1, 1, 1, 1, 1)$ .

where  $x$  labels the ordinary four coordinates,  $y$  the  $\delta$  extra coordinates,  $g$  refers to the determinant of the  $D$ -dimensional metric whose Ricci scalar is defined by  $\mathcal{R}$ , and  $\bar{M}_D$  is called the reduced Planck scale of the  $D$ -dimensional theory. In the second term of Eq. (84.7), which gives the gravitational interactions of SM fields, the  $D$ -dimensional metric reduces to the induced metric on the 3-brane where the SM fields propagate. The extra dimensions are assumed to be flat and compactified in a volume  $V_\delta$ . As an example, consider a toroidal compactification of equal radii  $R$  and volume  $V_\delta = (2\pi R)^\delta$ . After a KK reduction, one finds that the fields that couple to the SM are the spin-2 gravitational field  $G_{\mu\nu}(x, y)$  and a tower of spin-0 KK graviscalars [19]. The graviscalars, however, only couple to SM fields through the trace of the energy-momentum tensor, resulting in weaker couplings to the SM fields. The Fourier expansion of the spin-2 field is given by

$$G_{\mu\nu}(x, y) = G_{\mu\nu}^{(0)}(x) + \frac{1}{\sqrt{V_\delta}} \sum_{\vec{n} \neq 0} e^{i\vec{n} \cdot \vec{y}/R} G_{\mu\nu}^{(\vec{n})}(x), \quad (84.8)$$

where  $\vec{y} = (y_1, y_2, \dots, y_\delta)$  are the extra-dimensional coordinates and  $\vec{n} = (n_1, n_2, \dots, n_\delta)$ . Eq. (84.8) contains a massless state, the 4D graviton  $G_{\mu\nu}^{(0)}$ , and its KK tower  $G_{\mu\nu}^{(\vec{n})}$  with masses  $m_n^2 = |\vec{n}|^2/R^2$ . At energies below  $1/R$  the action is that of the zero mode

$$S_4^{(0)} = -\frac{\bar{M}_D^{2+\delta}}{2} \int d^4x V_\delta \sqrt{-g^{(0)}} \mathcal{R}^{(0)} + \int d^4x \sqrt{-g_{\text{ind}}^{(0)}} \mathcal{L}_{\text{SM}}, \quad (84.9)$$

where we can identify the 4D reduced Planck mass,  $M_P \equiv 1/\sqrt{8\pi G_N} \simeq 2.4 \times 10^{18}$  GeV, as a function of the  $D$ -dimensional parameters:

$$M_P^2 = V_\delta \bar{M}_D^{2+\delta} \equiv R^\delta M_D^{2+\delta}. \quad (84.10)$$

Fixing  $M_D$  at around the electroweak scale  $M_D \sim \text{TeV}$  to avoid introducing a new mass scale in the model, Eq. (84.10) gives a prediction for  $R$ :

$$\delta = 1, 2, \dots, 6 \rightarrow R \sim 10^9 \text{ km}, 0.5 \text{ mm}, \dots, 0.1 \text{ MeV}^{-1}. \quad (84.11)$$

The option  $\delta = 1$  is clearly ruled out, as it leads to modifications of Newton's law at solar system distances. However this is not the case for  $\delta \geq 2$ , and possible observable consequences can be sought in present and future experiments.

Consistency of the model requires a stabilization mechanism for the radii of the extra dimensions, to the values shown in Eq. (84.11). The fact that we need  $R \gg 1/M_D$  leads to a new hierarchy problem, the solution of which might require imposing supersymmetry in the extra-dimensional bulk (for the case of two extra dimensions see for example [20]).

### 84.2.2 Tests of the Gravitational Force Law at Sub-mm Distances

The KK modes of the graviton give rise to deviations from Newton's law of gravitation for distances  $\lesssim R$ . Such deviations are usually parameterized by a modified Newtonian potential of the form

$$V(r) = -G_N \frac{m_1 m_2}{r} \left[ 1 + \alpha e^{-r/\lambda} \right]. \quad (84.12)$$

For a 2-torus compactification,  $\alpha = 16/3$  and  $\lambda = R$ . Searches for deviations from Newton's law of gravitation have been performed in several experiments [21–24]. From Ref. [23] we have the constraint  $R < 30\mu\text{m}$  at 95% CL for  $\delta = 2$ , corresponding to  $M_D > 4.0$  TeV. We see then that bounds from Newton's law deviations are already pushing the scale  $M_D$  beyond the TeV for two extra dimensions.

### 84.2.3 Astrophysical and Cosmological Constraints

The light KK gravitons could be copiously produced in stars, carrying away energy. Ensuring that the graviton luminosity is low enough to preserve the agreement of stellar models with observations provides powerful bounds on the scale  $M_D$ . The most stringent bound arises from supernova SN1987A, giving  $M_D > 27$  (2.4) TeV for  $\delta = 2$  (3) [25]. After a supernova explosion, most of the KK gravitons stay gravitationally trapped in the remnant neutron star. The requirement that neutron stars are not excessively heated by KK decays into photons leads to  $M_D > 1700$  (76) TeV for  $\delta = 2$  (3) [26].

Cosmological constraints are also quite stringent [27]. To avoid overclosure of the Universe by relic gravitons one needs  $M_D > 7$  TeV for  $\delta = 2$ . Relic KK gravitons decaying into photons contribute to the cosmic diffuse gamma radiation, from which one can derive the bound  $M_D > 100$  TeV for  $\delta = 2$ .

We must mention however that bounds coming from the decays of KK gravitons into photons can be reduced if we assume that KK gravitons decay mainly into other non-SM states. This could happen, for example, if there were other 3-branes with hidden sectors residing on them [15].

### 84.2.4 Collider Signals

#### 84.2.4.1 Graviton and Other Particle Production

Although each KK graviton has a purely gravitational coupling, suppressed by  $1/M_P$ , inclusive processes in which one sums over the almost continuous spectrum of available gravitons have cross sections suppressed only by powers of  $M_D$ . Processes involving gravitons are therefore detectable in collider experiments if  $M_D \sim$  TeV. A number of experimental searches for evidence of large extra dimensions have been performed at colliders, and interpreted in the context of the ADD model.

One signature arises from direct graviton emission. By making a derivative expansion of Einstein gravity, one can construct an effective theory, valid for energies much lower than  $M_D$ , and use it to make predictions for graviton-emission processes at colliders [19, 28, 29]. Gravitons produced in the final state would escape detection, giving rise to missing transverse momentum ( $p_T^{\text{miss}}$ ). The results quoted below are 95% CL lower limits on  $M_D$  for a range of values of  $\delta$  between 2 and 6, with more stringent limits corresponding to lower  $\delta$  values.

At hadron colliders, experimentally sensitive channels include the jet ( $j$ ) +  $p_T^{\text{miss}}$  and  $\gamma$  +  $p_T^{\text{miss}}$  final states. ATLAS (CMS)  $j$  +  $p_T^{\text{miss}}$  results with 139 (137)  $\text{fb}^{-1}$  of 13 TeV data provide limits of  $M_D > 5.9 - 11.2$  TeV [30] ( $M_D > 5.5 - 10.7$  TeV [31]). For these analyses, both experiments are assuming leading order cross sections. Since the effective theory is only valid for energies much less than  $M_D$ , the results are quoted for the full space, and include the information that suppressing the graviton cross section by a factor  $M_D^4/\hat{s}^2$  for  $\sqrt{\hat{s}} > M_D$ , where  $\sqrt{\hat{s}}$  is the parton-level center-of-mass energy of the hard collision, weakens the limits on  $M_D$  by a negligible amount for  $\delta = 2$  ( $\sim 3\%$  for  $\delta = 6$ ). Less stringent limits are obtained by CMS [32] from analysis of 36  $\text{fb}^{-1}$  of 13 TeV data in the  $\gamma$  +  $p_T^{\text{miss}}$  final state ( $M_D > 2.85 - 2.90$  TeV for  $\delta = 3 - 6$ ). The analogous ATLAS search [33] uses full Run 2 statistics but does not quote ADD interpretation of the results.

In models in which the ADD scenario is embedded in a string theory at the TeV scale [18], we expect the string scale  $M_s$  to be smaller than  $M_D$ , and therefore expect production of string resonances at the LHC [34]. A result from CMS analyzing the dijet invariant mass distribution for 137  $\text{fb}^{-1}$  of 13 TeV data excludes string resonances that decay predominantly to  $q + g$  with masses below 7.9 TeV [35]. ATLAS dijet analysis [36] provides their results in the context of model-independent limits on the cross section times acceptance for generic resonances of a variety of possible widths, from which one can deduce similar lower mass limits  $\sim 8$  TeV for string resonances decaying to  $q + g$ .

84.2.4.2 *Virtual graviton effects*

One can also search for virtual graviton effects, the calculation of which however depends on the ultraviolet cut-off of the theory and is therefore very model dependent. In the literature, several different formulations exist [19, 29, 37] for the dimension-eight operator for gravity exchange at tree level:

$$\mathcal{L}_8 = \pm \frac{4}{M_{TT}^4} \left( T_{\mu\nu} T^{\mu\nu} - \frac{1}{\delta + 2} T_\mu^\mu T_\nu^\nu \right), \quad (84.13)$$

where  $T_{\mu\nu}$  is the energy-momentum tensor and  $M_{TT}$  is related to  $M_D$  by some model-dependent coefficient [38]. The relations with the parametrization of Refs. [37] and [19] are, respectively,  $M_{TT} = M_S$  and  $M_{TT} = (2/\pi)^{1/4} \Lambda_T$ . The experimental results below are given as 95% CL lower limits on  $M_{TT}$ , including in some cases the possibility of both constructive or destructive interference, depending on the sign chosen in Eq. (9).

The most stringent limits arise from LHC analyses of the dijet angular distribution. Using 35.9 fb<sup>-1</sup> of 13 TeV data, CMS [39] obtains results that correspond to an approximate limit of  $M_{TT} \gtrsim 9$  TeV.

The next most restrictive results come from the analyses of diphoton ( $M_{TT} > 6.1$  TeV from ATLAS [40] and  $M_{TT} > 8.3$  TeV from CMS [41]) and dilepton ( $M_{TT} > 6.5$  TeV from CMS [42]) mass spectra. The complete Run 2 (139 fb<sup>-1</sup>) analysis of ATLAS di-lepton data [43] does not quote the limits on the ADD model.

At the one-loop level, gravitons can also generate dimension-six operators with coefficients that are also model dependent. Experimental bounds on these operators can also give stringent constraints on  $M_D$  [38].

84.2.4.3 *Black Hole Production*

The physics at energies  $\sqrt{s} \sim M_D$  is sensitive to the details of the unknown quantum theory of gravity. Nevertheless, in the transplanckian regime,  $\sqrt{s} \gg M_D$ , one can rely on a semiclassical description of gravity to obtain predictions. An interesting feature of transplanckian physics is the creation of black holes [44, 45] (for a review see, for example, [46]). A black hole is expected to be formed in a collision in which the impact parameter is smaller than the Schwarzschild radius [47]:

$$R_S = \frac{1}{M_D} \left[ \frac{2^\delta \pi^{(\delta-3)/2}}{\delta + 2} \Gamma\left(\frac{\delta + 3}{2}\right) \frac{M_{BH}}{M_D} \right]^{1/(\delta+1)}, \quad (84.14)$$

where  $M_{BH}$  is the mass of the black hole, which would roughly correspond to the total energy in the collision. The cross section for black hole production can be estimated to be of the same order as the geometric area  $\sigma \sim \pi R_S^2$ . For  $M_D \sim \text{TeV}$ , this gives a production of  $\sim 10^7$  black holes at the  $\sqrt{s} = 14$  TeV LHC with an integrated luminosity of 30 fb<sup>-1</sup> [44, 45]. A black hole would provide a striking experimental signature since it is expected to thermally radiate with a Hawking temperature  $T_H = (\delta + 1)/(4\pi R_S)$ , and therefore would evaporate democratically into all SM states. Nevertheless, given the present constraints on  $M_D$ , the LHC will not be able to reach energies much above  $M_D$ . This implies that predictions based on the semiclassical approximation could receive sizable modifications from model-dependent quantum-gravity effects.

The most stringent limits on microscopic black holes arise from LHC searches which observed no excesses above the SM background in high-multiplicity final states. The results are usually quoted as model-independent limits on the cross section for new physics in the final state and kinematic region analyzed. These results can then be used to provide constraints of models of low-scale gravity and weakly-coupled string theory. In addition, limits are sometimes quoted on particular implementations of models, which are used as benchmarks to illustrate the sensitivity.

A CMS analysis [48] of multi-object final states using 36 fb<sup>-1</sup> of 13 TeV data, excludes semiclassical black holes below masses of up to 10.1 TeV for  $M_D = 2$  TeV and  $\delta = 6$ . Analogous Run 2 ATLAS analysis [49], using 3.0 fb<sup>-1</sup> of 13 TeV data, excludes black hole masses up to 9.0–9.7 TeV, depending on  $M_D$ , for  $\delta = 6$ . Another ATLAS search [50] for an excess of events with multiple high transverse momentum objects, including charged leptons and jets, using 3.2 fb<sup>-1</sup> of 13 TeV data, excludes semiclassical black holes below masses of  $\sim 8.7$  TeV for  $M_D = 2$  TeV and  $\delta = 6$ .

A complementary approach is to look for jet extinction at high transverse momenta, as we expect hard short distance scattering processes to be highly suppressed at energies above  $M_D$  [51]. The CMS analysis [52] of inclusive jet  $p_T$  spectrum in 10.7 fb<sup>-1</sup> of 8 TeV data set a lower limit of 3.3 TeV on the extinction mass scale.

For black hole masses near  $M_D$ , the semi-classical approximation is not valid, and one could instead expect quantum black holes (QBH) that decay primarily into two-body final states [53]. In the context of both ADD model with  $\delta = 6$  the QBHs have been searched in few final state analysis (based on 2.3 fb<sup>-1</sup> [54]), dijet [36, 39, 55], (same and different flavor) dilepton [56–60], photon+jet [61, 62] and lepton+jet [63] channels by both ATLAS and CMS experiments. The Run 2 results at 13 TeV provide lower limits on QBH masses of up-to 9.4 TeV in an ADD model with  $\delta = 6$ . The strongest constraints are from dijet searches.

In weakly-coupled string models the semiclassical description of gravity fails in the energy range between  $M_s$  and  $M_s/g_s^2$  where stringy effects are important. In this regime one expects, instead of black holes, the formation of string balls, made of highly excited long strings, that could be copiously produced at the LHC for  $M_s \sim$  TeV [64], and would evaporate thermally at the Hagedorn temperature giving rise to high-multiplicity events. The same analyses used to search for black holes can be interpreted in the context of string balls. For example, for the case of  $\delta = 6$  with  $M_s = M_D/1.26 = 3$  TeV, the ATLAS multiple high transverse momentum object analysis [49] excludes string balls with masses below 6.5 to 9.0 TeV for values of  $0.2 < g_s < 0.8$ . The CMS multi-object analysis [48] studies string ball production in two scenarios, both assuming  $\delta = 6$ . For the constant  $g_s = 0.2$  and  $1 < M_s < 3.5$  TeV the string ball masses below 7.2 to 9.4 TeV are excluded, while at constant  $M_s = 3.6$  TeV and  $0.2 < g_s < 0.4$  masses below 7.2 to 8.1 TeV are excluded.

### 84.3 TeV-Scale Extra Dimensions

#### 84.3.1 Warped Extra Dimensions

The RS model [3] is the most attractive setup of warped extra dimensions at the TeV scale, since it provides an alternative solution to the hierarchy problem. The RS model is based on a 5D theory with the extra dimension compactified in an orbifold,  $S^1/Z_2$ , a circle  $S^1$  with the extra identification of  $y$  with  $-y$ . This corresponds to the segment  $y \in [0, \pi R]$ , a manifold with boundaries at  $y = 0$  and  $y = \pi R$ . Let us now assume that this 5D theory has a cosmological constant in the bulk  $\Lambda$ , and on the two boundaries  $\Lambda_0$  and  $\Lambda_{\pi R}$ . The action is given by

$$S_5 = - \int d^4x dy \left\{ \sqrt{-g} \left[ \frac{1}{2} M_5^3 \mathcal{R} + \Lambda \right] + \sqrt{-g_0} \delta(y) \Lambda_0 + \sqrt{-g_{\pi R}} \delta(y - \pi R) \Lambda_{\pi R} \right\}, \quad (84.15)$$

where  $g_0$  and  $g_{\pi R}$  are the values of the determinant of the induced metric on the two respective boundaries. Einstein's equations can be solved, giving in this case the metric

$$ds^2 = a(y)^2 dx^\mu dx^\nu \eta_{\mu\nu} + dy^2, \quad a(y) = e^{-ky}, \quad (84.16)$$

where  $k = \sqrt{-\Lambda/6M_5^3}$ . Consistency of the solution requires  $\Lambda_0 = -\Lambda_{\pi R} = -\Lambda/k$ . The metric in Eq. (84.16) corresponds to a 5D AdS space. The factor  $a(y)$  is called the ‘‘warp’’ factor and

determines how 4D scales change as a function of the position in the extra dimension. In particular, this implies that energy scales for 4D fields localized at the boundary at  $y = \pi R$  are red-shifted by a factor  $e^{-k\pi R}$  with respect to those localized at  $y = 0$ . For this reason, the boundaries at  $y = 0$  and  $y = \pi R$  are usually referred to as the ultraviolet (UV) and infrared (IR) boundaries, respectively.

As in the ADD case, we can perform a KK reduction and obtain the low-energy effective theory of the 4D massless graviton. In this case we obtain

$$M_P^2 = \int_0^{\pi R} dy e^{-2ky} M_5^3 = \frac{M_5^3}{2k} \left(1 - e^{-2k\pi R}\right). \quad (84.17)$$

Taking  $M_5 \sim k \sim M_P$ , we can generate an IR-boundary scale of order  $ke^{-k\pi R} \sim \text{TeV}$  for an extra dimension of radius  $R \simeq 11/k$ . Mechanisms to stabilize  $R$  to this value have been proposed [65, 66] that, contrary to the ADD case, do not require introducing any new small or large parameter. Therefore a natural solution to the hierarchy problem can be achieved in this framework if the Higgs field, whose vacuum expectation value (VEV) is responsible for electroweak symmetry breaking, is localized at the IR-boundary where the effective mass scales are of order TeV. The radion field is generically heavy in models with a stabilized  $R$ . Nevertheless, it has been recently discussed that under some conditions a naturally light radion can arise [67–70]. In these cases the radion is identified with the dilaton, the Nambu-Goldstone boson associated to the spontaneous breaking of scale invariance, and its mass can be naturally below  $ke^{-k\pi R} \sim \text{TeV}$ .

In the RS model [3], all the SM fields were assumed to be localized on the IR-boundary. Nevertheless, for the hierarchy problem, only the Higgs field has to be localized there. SM gauge bosons and fermions can propagate in the 5D bulk [4–7] (for a review see, for example, [71, 72]). By performing a KK reduction from the 5D action of a gauge boson, we find [4, 5]

$$\frac{1}{g_4^2} = \int_0^{\pi R} dy \frac{1}{g_5^2} = \frac{\pi R}{g_5^2}, \quad (84.18)$$

where  $g_D$  ( $D = 4, 5$ ) is the gauge coupling in  $D$ -dimensions. Therefore the 4D gauge couplings can be of order one, as is the case of the SM, if one demands  $g_5^2 \sim \pi R$ . Using  $kR \sim 10$  and  $g_4 \sim 0.5$ , one obtains the 5D gauge coupling

$$g_5 \sim 4/\sqrt{k}. \quad (84.19)$$

Boundary kinetic terms for the gauge bosons can modify this relation, allowing for larger values of  $g_5\sqrt{k}$ .

Fermions propagating in a warped extra dimension have 4D massless zero-modes with wavefunctions which vary as  $f_0 \sim \exp[(1/2 - c_f)ky]$ , where  $c_fk$  is their 5D mass [7, 73]. Depending on the free parameter  $c_fk$ , fermions can be localized either towards the UV-boundary ( $c_f > 1/2$ ) or IR-boundary ( $c_f < 1/2$ ). Since the Higgs boson is localized on the IR-boundary, one can generate exponentially suppressed Yukawa couplings by having the fermion zero-modes localized towards the UV-boundary, generating naturally the light SM fermion spectrum [7]. A large overlap with the wavefunction of the Higgs is needed for the top quark, in order to generate its large mass, thus requiring it to be localized towards the IR-boundary. In conclusion, the large mass hierarchies present in the SM fermion spectrum can be easily obtained in warped models via suitable choices of the order-one parameters  $c_f$  [74]. In these scenarios, deviations in flavor physics from the SM predictions are expected to arise from flavor-changing KK gluon couplings [75], putting certain constraints on the parameters of the models and predicting new physics effects to be observed in  $B$ -physics processes (see, for example, [76, 77]).

The masses of the KK states can also be calculated. One finds [7]

$$m_n \simeq \left(n + \frac{\alpha}{2} - \frac{1}{4}\right) \pi k e^{-\pi k R}, \quad (84.20)$$

where  $n = 1, 2, \dots$  and  $\alpha = \{|c_f - 1/2|, 0, 1\}$  for KK fermions, KK gauge bosons and KK gravitons, respectively. Their masses are of order  $ke^{-\pi kR} \sim \text{TeV}$  (for this reason we refer to these scenarios as TeV-scale extra dimensions). The first KK state of the gauge bosons would be the lightest, while gravitons are expected to be the heaviest.

#### 84.3.1.1 Models of Electroweak Symmetry Breaking

Theories in warped extra dimensions can be used to implement symmetry breaking at low energies by boundary conditions (for a review see, for example, [78]). For example, for a  $U(1)$  gauge symmetry in the 5D bulk, this can be easily achieved by imposing a Dirichlet boundary condition on the IR-boundary for the gauge-boson field,  $A_\mu|_{y=\pi R} = 0$ . This makes the zero-mode gauge boson get a mass, given by  $m_A = g_4 \sqrt{2k/g_5^2} e^{-\pi kR}$ . A very different situation occurs if the Dirichlet boundary condition is imposed on the UV-boundary,  $A_\mu|_{y=0} = 0$ . In this case the zero-mode gauge boson disappears from the spectrum. Finally, if a Dirichlet boundary condition is imposed on the two boundaries, one obtains a massless 4D scalar corresponding to the fifth component of the 5D gauge boson,  $A_5$ . Thus, different scenarios can be implemented by appropriately choosing the 5D bulk gauge symmetry,  $\mathcal{G}_5$ , and the symmetries to which it reduces on the UV and IR-boundary,  $\mathcal{H}_{UV}$  and  $\mathcal{H}_{IR}$ , respectively. In all cases the KK spectrum comes in representations of the group  $\mathcal{G}_5$ .

Among the most interesting scenarios are those called gauge-Higgs unified models, where the Higgs boson appears as the fifth component of a 5D gauge boson,  $A_5$ . The Higgs mass is protected by the 5D gauge invariance and can only get a nonzero value from non-local one-loop effects [79]. To guarantee the relation  $M_{W'}^2 \simeq M_Z^2 \cos^2 \theta_W$ , a custodial  $SU(2)_V$  symmetry is needed in the bulk and IR-boundary [80]. The simplest realization [81, 82] has

$$\begin{aligned}\mathcal{G}_5 &= SU(3)_c \times SO(5) \times U(1)_X, \\ \mathcal{H}_{IR} &= SU(3)_c \times SO(4) \times U(1)_X, \\ \mathcal{H}_{UV} &= G_{SM}.\end{aligned}$$

The Higgs boson gets a potential at the one-loop level that triggers a VEV, breaking the electroweak symmetry. In these models there is a light Higgs boson whose mass can be around 125 GeV, as required by the discovered Higgs boson [83]. This state, as will be explained in Sec. 84.3.2, behaves as a composite pseudo-Nambu-Goldstone boson with couplings that deviate from the SM Higgs [84]. The present experimental determination of the Higgs couplings at the LHC, that agrees with the SM predictions, put important constraints on these scenarios [83]. The lightest KK modes of the model are color fermions with charges  $Q = -1/3, 2/3$  and  $5/3$  [85].

#### 84.3.1.2 Constraints from Electroweak Precision Tests

Models in which the SM gauge bosons propagate in 1/TeV-sized extra dimensions give generically large corrections to electroweak observables. When the SM fermions are confined on a boundary these corrections are universal and can be parameterized by four quantities:  $\hat{S}$ ,  $\hat{T}$ ,  $W$  and  $Y$ , as defined in Ref. [86]. For warped models, where the 5D gauge coupling of Eq. (84.19) is large, the most relevant parameter is  $\hat{T}$ , which gives the bound  $m_{KK} \gtrsim 10 \text{ TeV}$  [71]. When a custodial symmetry is imposed [80], the main constraint comes from the  $\hat{S}$  parameter, requiring  $m_{KK} \gtrsim 3 \text{ TeV}$ , independent of the value of  $g_5$ . Corrections to the  $Zb_L\bar{b}_L$  coupling can also be important [71], especially in warped models for electroweak symmetry breaking as the ones described above.

#### 84.3.1.3 Kaluza-Klein Searches

The main prediction of 1/TeV-sized extra dimensions is the presence of a discretized KK spectrum, with masses around the TeV scale, associated with the SM fields that propagate in the extra dimension.

In the RS model [3], only gravity propagates in the 5D bulk. Experimental searches have been performed for the lightest KK graviton through its decay to a variety of SM particle-antiparticle pairs. The results are usually interpreted in the plane of the dimensionless coupling  $k/M_P$  versus  $m_1$ , where  $M_P$  is the reduced Planck mass defined previously and  $m_1$  is the mass of the lightest KK excitation of the graviton. Since the AdS curvature  $\sim k$  cannot exceed the cut-off scale of the model, which is estimated to be  $\ell_5^{1/3} M_5$  [38], one must demand  $k \ll \sqrt{2\ell_5} M_P \simeq 40M_P$ . The most stringent limits currently arise from LHC searches for resonances in the dilepton and diphoton final states, using 13 TeV collisions. Searches with the  $\gamma\gamma$  final state are an especially powerful approach, given that these final states have a branching fraction twice that of any individual lepton flavor. The CMS analysis [41] of 138 fb<sup>-1</sup> of 13 TeV data excludes KK gravitons below 2.2 to 5.6 TeV, depending on the value of the coupling  $k/M_P$ , which is varied between 0.01 and 0.2, while ATLAS [87] using 139 fb<sup>-1</sup> and provides a lower limit on the KK graviton mass of 4.5 TeV for the coupling parameter 0.1. The CMS [42] dilepton analyses, combining results from the  $ee$  and  $\mu\mu$  channels, exclude KK gravitons with masses 2.47–4.78 TeV for  $k/M_P$  values of 0.01–0.1. The ATLAS [88] analysis of 139 fb<sup>-1</sup> of Run 2 data does not include a RS KK graviton interpretation of the results. Less stringent limits on the KK graviton mass can be derived from analyses of the dijet [35, 36, 89, 90],  $HH$  [91–100], and  $VV$  [101–105] final states, where  $V$  can represent either a  $W$  or  $Z$  boson.

In addition, both ATLAS and CMS experiments directly search for heavy radions, with masses above 1 TeV where the dominant decay mode is to pairs of bosons. The main production mechanism is gluon fusion. ATLAS [103] (CMS [106]) excludes radion masses below 3.2 TeV (3.1 TeV) for a radion decaying into  $WW, ZZ$ . Radions are also searched in final states with pairs of Higgs bosons  $HH$  [91, 94, 96, 97, 99, 107], and dijet final states [108] with additional gluons. Bounds for a light radion (1 keV–10 GeV mass range) can be found in [109].

In warped extra-dimensional models in which the SM fields propagate in the 5D bulk, the couplings of the KK graviton to  $ee/\mu\mu/\gamma\gamma$  are suppressed [110], and the above bounds do not apply. Furthermore, the KK graviton is the heaviest KK state (see Eq. (84.20)), and therefore experimental searches for KK gauge bosons and fermions are more appropriate discovery channels in these scenarios. For the scenarios discussed above in which only the Higgs boson and the top quark decays are localized close to the IR-boundary, the KK gauge bosons mainly decay into top quarks, longitudinal  $W/Z$  bosons, and Higgs bosons. Couplings to light SM fermions are suppressed by a factor  $g/\sqrt{g_5^2 k} \sim 0.2$  [7] for the value of Eq. (84.19) that is considered from now on. Searches have been made for evidence of the lightest KK excitation of the gluon, through its decay to  $t\bar{t}$  pairs. The searches take into account the natural KK gluon width, which is typically  $\sim 15\%$  of its mass. The decay of a heavy particle to  $t\bar{t}$  would tend to produce highly boosted top (anti-)quarks in the final state. Products of the subsequent hadronic top quark decay would therefore tend to be close to each other in the detector. In the case of  $t \rightarrow Wb \rightarrow jjb$  decays, the three jets could overlap with one another and not be individually reconstructed with the standard jet algorithms, while  $t \rightarrow Wb \rightarrow \ell\nu b$  decays could result in the lepton failing standard isolation requirements due to its proximity to the  $b$ -jet; in both cases, the efficiency for properly reconstructing the final state would fall as the mass of the original particle increases. To avoid the loss in sensitivity which would result, a number of techniques, known generally as top quark tagging [111, 112], have been developed to reconstruct and identify highly boosted top quarks, for example by using a single wide jet to contain all the decay products of a hadronic top decay. The large backgrounds from QCD jets can then be reduced by requiring the jet mass be consistent with that of a top quark, and also by examining the substructure of the wide jet for indication that it resulted from the hadronic decay of a top quark. These techniques are key to extending to very high masses the

range of accessible resonances decaying to  $t\bar{t}$  pairs. While the ATLAS search in  $139\text{ fb}^{-1}$  of Run 2 data [113] does not provide a KK interpretation, dedicated analysis from CMS [114] of  $36\text{ fb}^{-1}$  of 13 TeV data combines di-lepton, lepton-plus-jet, and all-hadronic  $t\bar{t}$  decays and excludes KK gluons with masses below 4.55 TeV. ATLAS uses all-hadronic [115] and lepton-plus-jet [116] final states to exclude KK gluons up to 3.4 and 3.8 TeV respectively with  $36\text{ fb}^{-1}$  of 13 TeV data. The results are not directly comparable between the two LHC experiments, since they employ in their respective analyses different implementations of the theoretical model. For masses between 3 and 5 TeV, the cross-section limits are around 20 fb for CMS analysis of  $36\text{ fb}^{-1}$  and 30 fb (4 fb) for ATLAS analyses of 36 (139)  $\text{fb}^{-1}$ .

A gauge boson KK excitation could be also sought through its decay to longitudinal  $W/Z$  bosons. Recent analyses from ATLAS [117] (and CMS [118]) with  $139$  ( $137$ )  $\text{fb}^{-1}$  of 13 TeV data searching for heavy vector resonances decaying to a  $W$  or  $Z$  boson and a Higgs in the  $q\bar{q}b\bar{b}$  final state have set a lower limit on the mass of these KK of 3.2 (3.7) TeV (warped models are equivalent to the Model B considered in the analyses with  $g_V \sim g_5\sqrt{k}$ ). The decay to a pair of intermediate vector bosons has also been exploited to search for KK gravitons in models in which the SM fields propagate in the 5D bulk. The analyses typically reconstruct hadronic  $W/Z$  decays using variants of the boosted techniques mentioned previously. An ATLAS analysis [119] combines leptonic and hadronic final states from the KK graviton decay  $G^* \rightarrow VV$ , where  $V$  can represent either a  $W$  or  $Z$  boson, exclude KK gravitons with masses below 2.3 TeV, for a value of  $k/M_P = 1$ . CMS  $VV$  analyses [106, 120, 121] using  $137\text{ fb}^{-1}$  of 13 TeV data also exclude KK gravitons with masses below 1.8 TeV in the context of bulk gravitons for a maximum value of  $k/M_P = 0.5$ .

Additionally, CMS has searched for the pair production of Standard Model Higgs bosons in the  $HH \rightarrow b\bar{b}b\bar{b}$  final state from the decay of warped extra-dimensional resonances. This has excluded KK gravitons with  $m_G < 1.2$  TeV under the narrow-width assumption. Similarly, for a radion, masses below 2.6 TeV (narrow) and 2.9 TeV (10% width) are excluded [122]. More generally the warped extra-dimensional interpretation in the CMS resonant- $HH$  combination [123] sets a lower limit on  $\Lambda_R \equiv \sqrt{6}e^{-\pi kR}M_P \approx 16$  TeV for radion masses near 1.2 TeV, and excludes  $k/M_P \gtrsim 0.3$  for KK graviton masses in the range  $0.3 < m_G < 1.5$  TeV.

CMS has also probed the topology in which a KK gluon ( $g_{KK}$ ) decays via an intermediate radion ( $R$ ) to three gluons ( $ggg$ ) [124]. The analysis reports upper limits on the model-independent quantity  $\sigma \times \mathcal{B}(g_{KK} \rightarrow gR \rightarrow ggg) \times \mathcal{A}$  as a function of the KK-gluon mass  $m_{g_{KK}}$  (and the mass ratio  $\rho_m \equiv m_R/m_{g_{KK}}$ ). A closely related CMS analysis targets the cascade  $g_{KK} \rightarrow gR \rightarrow gW^+W^-$  in the all-hadronic final state, using  $138\text{ fb}^{-1}$  [125]. It sets model-independent upper limits on  $\sigma \times \mathcal{B}(g_{KK} \rightarrow gR \rightarrow gWW) \times \mathcal{A}$  as functions of  $m_{g_{KK}}$  and  $m_R$ ; under the benchmark warped extra-dimensional couplings considered, exclusions extend up to  $m_{g_{KK}} \sim 3.5$  TeV with  $m_R \sim 2.2$  TeV.

The lightest KK states are, in certain models, the partners of the top quark. For example, in 5D composite Higgs models these are colored states with charges  $Q = -1/3, 2/3$  and  $5/3$  (arising from  $SU(2)_L$  doublets with  $Y = 7/6, 1/6$ ), and masses expected to be below the TeV [85]. They can be either singly or pair-produced, and mainly decay into a combination of  $W/Z$  with top/bottom quarks [126–129]. An exhaustive review of these searches can be found in Ref. [130]. Of particular note, the  $Q = 5/3$  state decays mainly into  $W^+t \rightarrow W^+W^+b$ , giving a very clean signature of a pair of same-sign leptons in the final state. An analysis by ATLAS [131] searching in the lepton-plus-jets final state for evidence of pair production of the  $Q = 5/3$  state provides a lower mass limit of 1.25 TeV. A CMS analysis [132] searching for pair production of the  $Q = 5/3$  state using both lepton-plus-jets and same sign lepton final states excludes masses below 1.3 TeV. Similarly, searches for single production of the  $Q = 5/3$  state [133], also excludes masses up to 0.9 to 1.5 TeV depending on the model parameters. Both LHC experiments have searched for pair production of vector-like quarks  $T$  and  $B$  of charges  $Q = 2/3$  and  $-1/3$  respectively, assuming the allowable decays are

$T \rightarrow Wb/Zt/Ht$  and  $B \rightarrow Wt/Zb/Hb$ . In each case, it is assumed the branching fractions of the three decay modes sum to unity, but the individual branching fractions, which are model-dependent, are allowed to vary within this constraint. Both ATLAS [134–136] and CMS [137, 138] obtain lower limits on the mass of the  $T$  and  $B$  vector-like quarks up to 1.5 TeV and 1.6 TeV respectively. A comprehensive review of such CMS searches can be found in Ref. [139].

Analyses from ATLAS [140–144] and CMS [133, 145–148] also search for a single top partner and single bottom partner production, the cross section for which is model-dependent [149] but does not carry the kinematic penalty for producing two heavy objects.

### 84.3.2 Connection with Strongly Coupled Models via the AdS/CFT Correspondence

The AdS/CFT correspondence [8] provides a connection between warped extra-dimensional models and strongly-coupled theories in ordinary 4D. Although the exact connection is only known for certain cases, the AdS/CFT techniques have been very useful to obtain, at the qualitative level, a 4D holographic description of the various phenomena in warped extra-dimensional models [11, 72].

The connection goes as follows. The physics of the bulk AdS<sub>5</sub> models can be interpreted as that of a 4D CFT which is strongly coupled. The extra-dimensional coordinate  $y$  plays the role of the renormalization scale  $\mu$  of the CFT by means of the identification  $\mu \equiv ke^{-ky}$ . Therefore the UV-boundary corresponds in the CFT to a UV cut-off scale at  $\Lambda_{UV} = k \sim M_P$ , breaking explicitly conformal invariance, while the IR-boundary can be interpreted as a spontaneous breaking of the conformal symmetry at energies  $ke^{-k\pi R} \sim \text{TeV}$ . Fields localized on the UV-boundary are elementary fields external to the CFT, while fields localized on the IR-boundary and KK states corresponds to composite resonances of the CFT. Furthermore, local gauge symmetries in the 5D models,  $\mathcal{G}_5$ , correspond to global symmetries of the CFT, while the UV-boundary symmetry can be interpreted as a gauging of the subgroup  $\mathcal{H}_{UV}$  of  $\mathcal{G}_5$  in the CFT. Breaking gauge symmetries by IR-boundary conditions corresponds to the spontaneous breaking  $\mathcal{G}_5 \rightarrow \mathcal{H}_{IR}$  in the CFT at energies  $\sim ke^{-k\pi R}$ . Using this correspondence one can easily derive the 4D massless spectrum of the compactified AdS<sub>5</sub> models. One also has the identification  $k^3/M_5^3 \approx 16\pi^2/N^2$  and  $g_5^2 k \approx 16\pi^2/N^r$  ( $r = 1$  or  $2$  for CFT fields in the fundamental or adjoint representation of the gauge group), where  $N$  plays the role of the number of colors of the CFT. Therefore the weak-coupling limit in AdS<sub>5</sub> corresponds to a large- $N$  expansion in the CFT.

Following the above AdS/CFT dictionary one can understand the RS solution to the hierarchy problem from a 4D viewpoint. The equivalent 4D model is a CFT with a TeV mass gap and a Higgs boson emerging as a composite state. In the particular case where the Higgs is the fifth-component of the gauge-boson,  $A_5$  [150], this corresponds to models, similar to those proposed in Ref. [151–154], where the Higgs is a composite pseudo-Nambu-Goldstone boson arising from the spontaneous breaking  $\mathcal{G}_5 \rightarrow \mathcal{H}_{IR}$  in the CFT. The AdS/CFT dictionary tells us that KK states must behave as composite resonances. For example, if the SM gauge bosons propagate in the 5D bulk, the lowest KK  $SU(2)_L$ -gauge boson must have properties similar to those of the Techni-rho  $\rho_T$  [130] with a coupling to longitudinal  $W/Z$  bosons given by  $g_5\sqrt{k} \approx g_{\rho_T}$ , while the coupling to elementary fermions is  $g^2/\sqrt{g_5^2 k} \approx g^2 F_{\rho_T}/M_{\rho_T}$ .

Fermions in compactified AdS<sub>5</sub> also have a simple 4D holographic interpretation. The 4D massless mode described in Sec. 84.3.1 corresponds to an external fermion  $\psi_i$  linearly coupled to a fermionic CFT operator  $\mathcal{O}_i$ :  $\mathcal{L}_{\text{int}} = \lambda_i \bar{\psi}_i \mathcal{O}_i + h.c.$ . The dimension of the operator  $\mathcal{O}_i$  is related to the 5D fermion mass according to  $\text{Dim}[\mathcal{O}_i] = |c_f + 1/2| - 1$ . Therefore, by varying  $c_f$  one varies  $\text{Dim}[\mathcal{O}_i]$ , making the coupling  $\lambda_i$  irrelevant ( $c_f > 1/2$ ), marginal ( $c_f = 1/2$ ) or relevant ( $c_f < 1/2$ ). When irrelevant, the coupling is exponentially suppressed at low energies, and then the coupling of  $\psi_i$  to the CFT (and eventually to the composite Higgs) is very small. When relevant, the coupling grows in the IR and become as large as  $g_5$  (in units of  $k$ ), meaning that the fermion is as strongly

coupled as the CFT states [81]. In this latter case  $\psi_i$  behaves as a composite fermion.

### 84.3.3 *Linear dilaton geometry*

The warp factor  $a(y)$  in Eq. (84.16) can be different from the exponential considered above, giving rise to a different KK mass spectrum. A particularly interesting case is the linear dilaton geometry [155] where the KK states become very narrowly-spaced. These scenarios can also be understood in 4D via the clockwork mechanism [156], and for this reason are usually referred to as Clockwork/Linear Dilaton (CW/LD) models. The collider phenomenology becomes different in these models since the decay of the narrowly-spaced KK spectrum appears as a periodic signal in the invariant mass of the decaying products. An excellent detector resolution is needed in order to resolve these KK modes which can be done by looking at 2 electrons or 2 photons in the final state. A search for this type of KK gravitons decaying into  $ee$  [157] and  $\gamma\gamma$  [41, 157, 158] put bounds for  $M_5$  in the range of 10 TeV to 1 TeV for values of the lightest KK graviton mass (referred as  $k$  in [157, 158]) in the range of few TeVs.

### 84.3.4 *Flat Extra Dimensions*

Models with quantum gravity at the TeV scale, as in the ADD scenario, can have extra (flat) dimensions of  $1/\text{TeV}$  size, as happens in string scenarios (see, for example, [159]). All SM fields may propagate in these extra dimensions, leading to the possibility of observing their corresponding KK states.

A simple example is to assume that the SM gauge bosons propagate in a flat five-dimensional orbifold  $S^1/Z_2$  of radius  $R$ , with the fermions localized on a 4D boundary. The KK gauge bosons behave as sequential SM gauge bosons with a coupling to fermions enhanced by a factor  $\sqrt{2}$  [159]. The experimental limits on such sequential gauge bosons could therefore be recast as limits on KK gauge bosons. Such an interpretation of the ATLAS 7 TeV dilepton analysis [160] yielded the bound  $1/R > 4.16$  TeV, while a CMS 8 TeV search with a lepton and missing transverse energy in the final state [161] give  $1/R > 3.4$  TeV. Indirect bounds from LEP2 require however  $1/R \gtrsim 6$  TeV [86, 162], a bound that can considerably improve in the future by high-energy measurements of the dilepton invariant mass spectrum from Drell-Yan processes at the LHC [163]. More recent LHC limits on leptonically decaying gauge bosons [88, 164–168] are not interpreted as bounds on  $1/R$  by the collaborations, but the published results allow for independent derivation of such bounds.

An alternative scenario, known as Universal Extra Dimensions (UED) [169] (for a review see, for example, [170]), assumes that all SM fields propagate universally in a flat orbifold  $S^1/Z_2$  with an extra  $Z_2$  parity, called KK-parity, that interchanges the two boundaries. In this case, the lowest KK state is stable and is a Dark Matter candidate. At colliders, the KK particles would have to be created in pairs, and would then cascade decay to the lightest KK particle, which would be stable and escape detection. The UED mass-spectrum depends not only on the extra-dimensional radius  $R$ , but also on the cut-off of the 5D theory  $\Lambda$ , since quantum corrections sensitive to  $\Lambda R$  induce mass-splittings between the KK states. Experimental signatures, such as jets or leptons and  $p_T^{\text{miss}}$ , would be similar to those of typical  $R$ -parity conserving SUSY searches. An interpretation of the recent LHC experimental SUSY searches for UED models has been presented in Refs. [171, 172]. A lower bound  $1/R > 1.4 - 1.5$  TeV was derived for  $\Lambda R$  in the range  $5 - 35$  [171]. A recent analysis is given in Ref. [173] where it is shown that the minimal UED model is ruled out when LHC data is combined with Dark Matter relic density data. Extensions to the minimal UED model where boundary terms are included can however be compatible with experiments [173].

Finally, realistic models of electroweak symmetry breaking can also be constructed with flat extra spatial dimensions, similarly to those in the warped case, requiring, however, the presence of sizeable boundary kinetic terms [174]. There is also the possibility of breaking supersymmetry by boundary conditions [175]. Models of this type could explain naturally the presence of a Higgs

boson lighter than  $M_D \sim \text{TeV}$  (see, for example, [176–178]).

### References

- [1] For a comprehensive collection of the original papers see, “Modern Kaluza-Klein Theories”, edited by T. Appelquist *et al.*, Addison-Wesley (1987).
- [2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429**, 263 (1998), [hep-ph/9803315].
- [3] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999), [hep-ph/9905221].
- [4] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, *Phys. Lett.* **B473**, 43 (2000), [hep-ph/9911262].
- [5] A. Pomarol, *Phys. Lett.* **B486**, 153 (2000), [hep-ph/9911294].
- [6] S. Chang *et al.*, *Phys. Rev.* **D62**, 084025 (2000), [hep-ph/9912498].
- [7] T. Gherghetta and A. Pomarol, *Nucl. Phys.* **B586**, 141 (2000), [hep-ph/0003129].
- [8] J. M. Maldacena, *Int. J. Theor. Phys.* **38**, 1113 (1999), [Adv. Theor. Math. Phys.2,231(1998)], [hep-th/9711200].
- [9] E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998), [hep-th/9802150].
- [10] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, *Phys. Lett.* **B428**, 105 (1998), [hep-th/9802109].
- [11] N. Arkani-Hamed, M. Porrati and L. Randall, *JHEP* **08**, 017 (2001), [hep-th/0012148].
- [12] ATLAS public results are available on WWW at <https://twiki.cern.ch/twiki/bin/view/AtlasPublic>.
- [13] CMS public results are available on WWW at <https://cms-results.web.cern.ch/cms-results/public-results/publications>.
- [14] Z. Chacko, M. A. Luty and E. Ponton, *JHEP* **07**, 036 (2000), [hep-ph/9909248].
- [15] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *Phys. Rev.* **D59**, 086004 (1999), [hep-ph/9807344].
- [16] R. Rattazzi, hep-ph/0607055 (2006); I. Antoniadis, Yellow report CERN-2002-002 (2002).
- [17] J. D. Lykken, *Phys. Rev.* **D54**, R3693 (1996), [hep-th/9603133].
- [18] I. Antoniadis *et al.*, *Phys. Lett.* **B436**, 257 (1998), [hep-ph/9804398].
- [19] G. F. Giudice, R. Rattazzi and J. D. Wells, *Nucl. Phys.* **B544**, 3 (1999), [hep-ph/9811291].
- [20] N. Arkani-Hamed *et al.*, *Phys. Rev.* **D62**, 105002 (2000), [hep-ph/9912453].
- [21] E. G. Adelberger *et al.*, *Prog. Part. Nucl. Phys.* **62**, 102 (2009).
- [22] J. Murata and S. Tanaka, *Class. Quant. Grav.* **32**, 3, 033001 (2015), [arXiv:1408.3588].
- [23] W.-H. Tan *et al.*, *Phys. Rev. Lett.* **116**, 13, 131101 (2016).
- [24] J. G. Lee *et al.*, *Phys. Rev. Lett.* **124**, 101101 (2020), [arXiv:2002.11761].
- [25] C. Hanhart *et al.*, *Phys. Lett.* **B509**, 1 (2001), [arXiv:astro-ph/0102063].
- [26] S. Hannestad and G. G. Raffelt, *Phys. Rev.* **D67**, 125008 (2003), [Erratum: *Phys. Rev.*D69,029901(2004)], [hep-ph/0304029].
- [27] L. J. Hall and D. Tucker-Smith, *Phys. Rev.* **D60**, 085008 (1999), [hep-ph/9904267].
- [28] E. A. Mirabelli, M. Perelstein and M. E. Peskin, *Phys. Rev. Lett.* **82**, 2236 (1999), [hep-ph/9811337].
- [29] T. Han, J. D. Lykken and R.-J. Zhang, *Phys. Rev.* **D59**, 105006 (1999), [hep-ph/9811350].
- [30] G. Aad *et al.* (ATLAS), *Phys. Rev. D* **103**, 112006 (2021), [arXiv:2102.10874].

- [31] A. Tumasyan *et al.* (CMS), *JHEP* **11**, 153 (2021), [arXiv:2107.13021].
- [32] A. M. Sirunyan *et al.* (CMS), *JHEP* **02**, 074 (2019), [arXiv:1810.00196].
- [33] G. Aad *et al.* (ATLAS), *JHEP* **02**, 226 (2021), [arXiv:2011.05259].
- [34] S. Cullen, M. Perelstein and M. E. Peskin, *Phys. Rev.* **D62**, 055012 (2000), [hep-ph/0001166].
- [35] A. M. Sirunyan *et al.* (CMS), *JHEP* **05**, 033 (2020), [arXiv:1911.03947].
- [36] G. Aad *et al.* (ATLAS), *JHEP* **03**, 145 (2020), [arXiv:1910.08447].
- [37] J. L. Hewett, *Phys. Rev. Lett.* **82**, 4765 (1999), [hep-ph/9811356].
- [38] G. F. Giudice and A. Strumia, *Nucl. Phys.* **B663**, 377 (2003), [hep-ph/0301232].
- [39] A. M. Sirunyan *et al.* (CMS), *Eur. Phys. J.* **C78**, 9, 789 (2018), [arXiv:1803.08030].
- [40] M. Aaboud *et al.* (ATLAS), *Phys. Lett.* **B775**, 105 (2017), [arXiv:1707.04147].
- [41] A. Hayrapetyan *et al.* (CMS), *JHEP* **08**, 215 (2024), [arXiv:2405.09320].
- [42] A. M. Sirunyan *et al.*, *JHEP* **07**, 7 (2021), [arXiv:2103.02708].
- [43] G. Aad *et al.*, *JHEP* **11**, 005 (2020), [arXiv:2006.12946].
- [44] S. B. Giddings and S. D. Thomas, *Phys. Rev.* **D65**, 056010 (2002), [hep-ph/0106219].
- [45] S. Dimopoulos and G. L. Landsberg, *Phys. Rev. Lett.* **87**, 161602 (2001), [hep-ph/0106295].
- [46] P. Kanti, *Int. J. Mod. Phys.* **A19**, 4899 (2004), [hep-ph/0402168].
- [47] R. C. Myers and M. J. Perry, *Annals Phys.* **172**, 304 (1986).
- [48] A. M. Sirunyan *et al.* (CMS), *JHEP* **11**, 042 (2018), [arXiv:1805.06013].
- [49] G. Aad *et al.* (ATLAS), *JHEP* **03**, 026 (2016), [arXiv:1512.02586].
- [50] M. Aaboud *et al.* (ATLAS), *Phys. Lett.* **B760**, 520 (2016), [arXiv:1606.02265].
- [51] C. Kilic *et al.*, *Phys. Rev.* **D89**, 1, 016003 (2014), [arXiv:1207.3525].
- [52] V. Khachatryan *et al.* (CMS), *Phys. Rev.* **D90**, 3, 032005 (2014), [arXiv:1405.7653].
- [53] P. Meade and L. Randall, *JHEP* **05**, 003 (2008), [arXiv:0708.3017].
- [54] A. M. Sirunyan *et al.* (CMS), *Phys. Lett.* **B774**, 279 (2017), [arXiv:1705.01403].
- [55] G. Aad *et al.* (ATLAS), *Phys. Lett. B* **754**, 302 (2016), [arXiv:1512.01530].
- [56] G. Aad *et al.* (ATLAS) (2023), [arXiv:2307.08567].
- [57] M. Aaboud *et al.* (ATLAS), *Eur. Phys. J. C* **76**, 10, 541 (2016), [arXiv:1607.08079].
- [58] M. Aaboud *et al.* (ATLAS), *Phys. Rev.* **D98**, 9, 092008 (2018), [arXiv:1807.06573].
- [59] A. Tumasyan *et al.* (CMS), *JHEP* **05**, 227 (2023), [arXiv:2205.06709].
- [60] A. M. Sirunyan *et al.* (CMS), *JHEP* **04**, 073 (2018), [arXiv:1802.01122].
- [61] A. Tumasyan *et al.* (CMS) (2023), [arXiv:2305.07998].
- [62] M. Aaboud *et al.* (ATLAS), *Eur. Phys. J. C* **78**, 2, 102 (2018), [arXiv:1709.10440].
- [63] G. Aad *et al.* (ATLAS), *Phys. Rev. D* **109**, 3, 032010 (2024), [arXiv:2307.14967].
- [64] S. Dimopoulos and R. Emparan, *Phys. Lett.* **B526**, 393 (2002), [hep-ph/0108060].
- [65] W. D. Goldberger and M. B. Wise, *Phys. Rev. Lett.* **83**, 4922 (1999), [hep-ph/9907447].
- [66] J. Garriga and A. Pomarol, *Phys. Lett.* **B560**, 91 (2003), [hep-th/0212227].
- [67] See talk by R. Rattazzi at Planck 2010, CERN, <https://indico.cern.ch/event/75810/contributions/1250635/>.
- [68] B. Bellazzini *et al.*, *Eur. Phys. J.* **C74**, 2790 (2014), [arXiv:1305.3919].

- [69] F. Coradeschi *et al.*, *JHEP* **11**, 057 (2013), [arXiv:1306.4601].
- [70] E. Megias and O. Pujolas, *JHEP* **08**, 081 (2014), [arXiv:1401.4998].
- [71] E. P. H. Davoudiasl, S. Gopalakrishna and J. Santiago, *New J. Phys.* **12**, 075011 (2010), [arXiv:0908.1968].
- [72] T. Gherghetta, in “Physics of the large and the small, TASI 09, proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, USA, 1-26 June 2009,” 165–232 (2011), [arXiv:1008.2570].
- [73] Y. Grossman and M. Neubert, *Phys. Lett.* **B474**, 361 (2000), [hep-ph/9912408].
- [74] S. J. Huber and Q. Shafi, *Phys. Lett.* **B498**, 256 (2001), [hep-ph/0010195].
- [75] A. Delgado, A. Pomarol and M. Quiros, *JHEP* **01**, 030 (2000), [hep-ph/9911252].
- [76] K. Agashe, G. Perez and A. Soni, *Phys. Rev.* **D71**, 016002 (2005), [hep-ph/0408134].
- [77] M. Bauer *et al.*, *JHEP* **09**, 017 (2010), [arXiv:0912.1625].
- [78] A. Pomarol, *Int. J. Mod. Phys.* **A24**, 61 (2009), [ In Kane, Gordon (ed.) *et al.: Perspectives on LHC physics* ,259(2008)].
- [79] Y. Hosotani, *Phys. Lett.* **126B**, 309 (1983).
- [80] K. Agashe *et al.*, *JHEP* **08**, 050 (2003), [hep-ph/0308036].
- [81] K. Agashe, R. Contino and A. Pomarol, *Nucl. Phys.* **B719**, 165 (2005), [hep-ph/0412089].
- [82] For a review see, for example, R. Contino, arXiv:1005.4269.
- [83] See, for example, PDG review of Higgs boson in this *Review*.
- [84] G. F. Giudice *et al.*, *JHEP* **06**, 045 (2007), [hep-ph/0703164].
- [85] R. Contino, L. Da Rold and A. Pomarol, *Phys. Rev.* **D75**, 055014 (2007), [hep-ph/0612048].
- [86] R. Barbieri *et al.*, *Nucl. Phys.* **B703**, 127 (2004), [hep-ph/0405040].
- [87] G. Aad *et al.* (ATLAS), *Phys. Lett. B* **822**, 136651 (2021), [arXiv:2102.13405].
- [88] G. Aad *et al.* (ATLAS), *Phys. Lett.* **B796**, 68 (2019), [arXiv:1903.06248].
- [89] A. M. Sirunyan *et al.* (CMS), *JHEP* **08**, 130 (2018), [arXiv:1806.00843].
- [90] G. Aad *et al.* (ATLAS), *Phys. Rev. Lett.* **125**, 13, 131801 (2020), [arXiv:2005.02983].
- [91] A. M. Sirunyan *et al.* (CMS), *Phys. Rev. Lett.* **122**, 12, 121803 (2019), [arXiv:1811.09689].
- [92] M. Aaboud *et al.* (ATLAS), *JHEP* **01**, 030 (2019), [arXiv:1804.06174].
- [93] G. Aad *et al.* (ATLAS), *Phys. Lett. B* **800**, 135103 (2020), [arXiv:1906.02025].
- [94] G. Aad *et al.* (ATLAS), *Phys. Rev. D* **106**, 5, 052001 (2022), [arXiv:2112.11876].
- [95] G. Aad *et al.* (ATLAS), *JHEP* **11**, 163 (2020), [arXiv:2007.14811].
- [96] A. M. Sirunyan *et al.* (CMS), *Phys. Rev. D* **102**, 3, 032003 (2020), [arXiv:2006.06391].
- [97] A. Tumasyan *et al.* (CMS), *JHEP* **05**, 005 (2022), [arXiv:2112.03161].
- [98] A. Tumasyan *et al.* (CMS), *JHEP* **11**, 057 (2021), [arXiv:2106.10361].
- [99] A. Tumasyan *et al.* (CMS), *JHEP* **07**, 095 (2023), [arXiv:2206.10268].
- [100] A. Hayrapetyan *et al.* (CMS), *JHEP* **07**, 293 (2024), [arXiv:2403.09430].
- [101] G. Aad *et al.* (ATLAS), *JHEP* **09**, 091 (2019), [Erratum: *JHEP* **06**, 042 (2020)], [arXiv:1906.08589].
- [102] A. M. Sirunyan *et al.* (CMS), *Eur. Phys. J. C* **80**, 3, 237 (2020), [arXiv:1906.05977].
- [103] G. Aad *et al.* (ATLAS), *Eur. Phys. J. C* **80**, 12, 1165 (2020), [arXiv:2004.14636].

- [104] G. Aad *et al.* (ATLAS), *Eur. Phys. J. C* **81**, 4, 332 (2021), [arXiv:2009.14791].
- [105] A. Tumasyan *et al.* (CMS), *Phys. Lett. B* **844**, 137813 (2023), [arXiv:2210.00043].
- [106] A. Tumasyan *et al.* (CMS), *Phys. Rev. D* **105**, 3, 032008 (2022), [arXiv:2109.06055].
- [107] A. M. Sirunyan *et al.* (CMS), *JHEP* **01**, 051 (2019), [arXiv:1808.01365].
- [108] A. Tumasyan *et al.* (CMS), *Phys. Lett. B* **832**, 137263 (2022), [arXiv:2201.02140].
- [109] F. Abu-Ajamieh, J. S. Lee and J. Terning, *JHEP* **10**, 050 (2018), [arXiv:1711.02697].
- [110] K. Agashe *et al.*, *Phys. Rev.* **D76**, 036006 (2007), [hep-ph/0701186].
- [111] M. Aaboud *et al.* (ATLAS), *Eur. Phys. J. C* **79**, 5, 375 (2019), [arXiv:1808.07858].
- [112] A. M. Sirunyan *et al.* (CMS), *JINST* **15**, 06, P06005 (2020), [arXiv:2004.08262].
- [113] G. Aad *et al.* (ATLAS), *JHEP* **10**, 061 (2020), [arXiv:2005.05138].
- [114] A. M. Sirunyan *et al.* (CMS), *JHEP* **04**, 031 (2019), [arXiv:1810.05905].
- [115] M. Aaboud *et al.* (ATLAS), *Phys. Rev.* **D99**, 9, 092004 (2019), [arXiv:1902.10077].
- [116] M. Aaboud *et al.* (ATLAS), *Eur. Phys. J.* **C78**, 7, 565 (2018), [arXiv:1804.10823].
- [117] G. Aad *et al.* (ATLAS), *Phys. Rev. D* **102**, 11, 112008 (2020), [arXiv:2007.05293].
- [118] A. M. Sirunyan *et al.* (CMS), *Eur. Phys. J. C* **81**, 8, 688 (2021), [arXiv:2102.08198].
- [119] M. Aaboud *et al.* (ATLAS), *Phys. Rev.* **D98**, 5, 052008 (2018), [arXiv:1808.02380].
- [120] A. Tumasyan *et al.* (CMS), *Phys. Rev. D* **106**, 1, 012004 (2022), [arXiv:2109.08268].
- [121] A. Tumasyan *et al.* (CMS), *JHEP* **04**, 087 (2022), [arXiv:2111.13669].
- [122] A. Tumasyan *et al.* (CMS), *JHEP* **02**, 040 (2025), [arXiv:2407.13872].
- [123] A. Hayrapetyan *et al.* (CMS), *Phys. Rept.* **1115**, 368 (2025), [arXiv:2403.16926].
- [124] A. Hayrapetyan *et al.* (CMS), *Phys. Rev. Lett.* **133**, 1, 011801 (2024), [arXiv:2310.14023].
- [125] A. Hayrapetyan *et al.* (CMS), *JHEP* **02**, 199 (2025), [arXiv:2410.17303].
- [126] R. Contino and G. Servant, *JHEP* **06**, 026 (2008), [arXiv:0801.1679].
- [127] J. A. Aguilar-Saavedra, *JHEP* **11**, 030 (2009), [arXiv:0907.3155].
- [128] J. Mrazek and A. Wulzer, *Phys. Rev.* **D81**, 075006 (2010), [arXiv:0909.3977].
- [129] G. Dissertori *et al.*, *JHEP* **09**, 019 (2010), [arXiv:1005.4414].
- [130] See, for example, PDG review of Technicolor searches in this volume.
- [131] M. Aaboud *et al.* (ATLAS), *JHEP* **10**, 141 (2017), [arXiv:1707.03347].
- [132] A. M. Sirunyan *et al.* (CMS), *JHEP* **03**, 082 (2019), [arXiv:1810.03188].
- [133] A. M. Sirunyan *et al.* (CMS), *Eur. Phys. J. C* **79**, 90 (2019), [arXiv:1809.08597].
- [134] G. Aad *et al.* (ATLAS), *Eur. Phys. J. C* **83**, 8, 719 (2023), [arXiv:2212.05263].
- [135] G. Aad *et al.* (ATLAS), *Phys. Lett. B* **843**, 138019 (2023), [arXiv:2210.15413].
- [136] M. Aaboud *et al.* (ATLAS), *Phys. Rev. Lett.* **121**, 21, 211801 (2018), [arXiv:1808.02343].
- [137] A. Tumasyan *et al.* (CMS), *JHEP* **07**, 020 (2023), [arXiv:2209.07327].
- [138] A. M. Sirunyan *et al.* (CMS), *Phys. Rev. D* **102**, 112004 (2020), [arXiv:2008.09835].
- [139] A. Hayrapetyan *et al.* (CMS), *Phys. Rept.* **1115**, 570 (2025), [arXiv:2405.17605].
- [140] G. Aad *et al.* (ATLAS), *JHEP* **11**, 168 (2023), [arXiv:2308.02595].
- [141] G. Aad *et al.* (ATLAS), *Phys. Rev. D* **109**, 11, 112012 (2024), [arXiv:2307.07584].
- [142] G. Aad *et al.* (ATLAS), *JHEP* **08**, 153 (2023), [arXiv:2305.03401].

- [143] G. Aad *et al.* (ATLAS), *Phys. Rev. D* **105**, 9, 092012 (2022), [arXiv:2201.07045].
- [144] M. Aaboud *et al.* (ATLAS), *JHEP* **05**, 164 (2019), [arXiv:1812.07343].
- [145] A. Tumasyan *et al.* (CMS), *JHEP* **09**, 057 (2023), [arXiv:2302.12802].
- [146] A. Tumasyan *et al.* (CMS), *JHEP* **05**, 093 (2022), [arXiv:2201.02227].
- [147] A. M. Sirunyan *et al.* (CMS), *JHEP* **01**, 036 (2020), [arXiv:1909.04721].
- [148] A. M. Sirunyan *et al.* (CMS), *JHEP* **06**, 031 (2018), [arXiv:1802.01486].
- [149] A. De Simone *et al.*, *JHEP* **04**, 004 (2013), [arXiv:1211.5663].
- [150] R. Contino, Y. Nomura and A. Pomarol, *Nucl. Phys.* **B671**, 148 (2003), [hep-ph/0306259].
- [151] D. B. Kaplan and H. Georgi, *Phys. Lett.* **136B**, 183 (1984).
- [152] D. B. Kaplan, H. Georgi and S. Dimopoulos, *Phys. Lett. B* **136**, 187 (1984).
- [153] T. Banks, *Nucl. Phys. B* **243**, 125 (1984).
- [154] H. Georgi, D. B. Kaplan and P. Galison, *Phys. Lett.* **143B**, 152 (1984).
- [155] I. Antoniadis *et al.*, *Phys. Rev. Lett.* **108**, 081602 (2012), [arXiv:1102.4043].
- [156] G. F. Giudice and M. McCullough, *JHEP* **02**, 036 (2017), [arXiv:1610.07962].
- [157] G. Aad *et al.* (ATLAS), *JHEP* **10**, 079 (2023), [arXiv:2305.10894].
- [158] A. M. Sirunyan *et al.* (CMS), *Phys. Rev. D* **98**, 9, 092001 (2018), [arXiv:1809.00327].
- [159] I. Antoniadis and K. Benakli, *Int. J. Mod. Phys.* **A15**, 4237 (2000), [hep-ph/0007226].
- [160] G. Aad *et al.* (ATLAS), *JHEP* **11**, 138 (2012), [arXiv:1209.2535].
- [161] V. Khachatryan *et al.* (CMS), *Phys. Rev.* **D91**, 9, 092005 (2015), [arXiv:1408.2745].
- [162] K. Cheung and G. L. Landsberg, *Phys. Rev.* **D65**, 076003 (2002), [hep-ph/0110346].
- [163] M. Farina *et al.*, *Phys. Lett.* **B772**, 210 (2017), [arXiv:1609.08157].
- [164] M. Aaboud *et al.* (ATLAS), *Eur. Phys. J. C* **78**, 5, 401 (2018), [arXiv:1706.04786].
- [165] G. Aad *et al.* (ATLAS), *Phys. Rev. D* **100**, 5, 052013 (2019), [arXiv:1906.05609].
- [166] A. M. Sirunyan *et al.* (CMS), *JHEP* **06**, 128 (2018), [arXiv:1803.11133].
- [167] A. M. Sirunyan *et al.* (CMS), *JHEP* **06**, 120 (2018), [arXiv:1803.06292].
- [168] A. Tumasyan *et al.* (CMS), *JHEP* **09**, 051 (2023), [arXiv:2212.12604].
- [169] T. Appelquist, H.-C. Cheng and B. A. Dobrescu, *Phys. Rev.* **D64**, 035002 (2001), [hep-ph/0012100].
- [170] A. Datta, K. Kong and K. T. Matchev, *New J. Phys.* **12**, 075017 (2010), [arXiv:1002.4624].
- [171] N. Deutschmann, T. Flacke and J. S. Kim, *Phys. Lett.* **B771**, 515 (2017), [arXiv:1702.00410].
- [172] J. Beuria *et al.*, *Comput. Phys. Commun.* **226**, 187 (2018), [arXiv:1702.00413].
- [173] Avnish *et al.*, *Phys. Rev. D* **103**, 115011 (2021), [arXiv:2012.15137].
- [174] G. Panico, M. Safari and M. Serone, *JHEP* **02**, 103 (2011), [arXiv:1012.2875].
- [175] J. Scherk and J. H. Schwarz, *Phys. Lett.* **82B**, 60 (1979).
- [176] A. Pomarol and M. Quiros, *Phys. Lett.* **B438**, 255 (1998), [hep-ph/9806263].
- [177] I. Antoniadis *et al.*, *Nucl. Phys.* **B544**, 503 (1999), [hep-ph/9810410].
- [178] R. Barbieri, L. J. Hall and Y. Nomura, *Phys. Rev.* **D63**, 105007 (2001), [hep-ph/0011311].