58. τ Branching Fractions

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58.1 τ Branching Fractions

The τ Listings contains 252 entries that correspond to either a τ partial decay fraction into a specific decay mode (branching fraction) or a ratio of two τ partial decay fractions (branching ratio). Experimental measurements provide values for 148 of these quantities, upper limits for 67 branching fractions to Lepton Family number, Lepton number, or Baryon number violating modes, and 37 additional upper limits for other modes. A total of 129 τ branching fractions and branching ratios are determined with a fit of 171 measurements. 85 quantities have at least one measurement in the fit.

58.2 The constrained fit to τ branching fractions

The τ branching fractions fit uses the reported values, uncertanties and statistical correlations of the τ branching fractions and branching ratios measurements. Asymmetric uncertainties are symmetrized as $\sigma^2_{\text{symm}} = (\sigma^2_+ + \sigma^2_-)/2$. If only a few measurements are correlated, the correlation coefficients are listed in the footnote for each measurement (see for example Γ(particle$^-$ $\geq$ 0 neutrals $\geq 0\,K^0\nu_\tau$ ("1-prong"))$/\Gamma_{\text{total}}$). If a large number of measurements are correlated, then the full correlation matrix is listed in the footnote to the measurement that first appears in the τ Listings. Footnotes to the other measurements refer to the first one. For example, the large correlation matrices for the branching fraction or ratio measurements contained in Refs. [1] [2] are listed in Footnotes to the $\Gamma(e^-\overline{\nu}_e\nu_\tau)/\Gamma_{\text{total}}$ and $\Gamma(h^-\nu_\tau)/\Gamma_{\text{total}}$ measurements respectively. Additionally, the most precise experimental inputs are treated according to how they depend on external parameters on the basis of their documentation [3]. The τ measurements may depend on parameters such as the τ pair production cross-section in e^+e^- annihilations at the $\Upsilon(4S)$ peak. In some cases, measurements reported in different papers by the same collaboration may depend on common parameters like the estimate of the integrated luminosity or of particle identification efficiencies. For all the significant detected dependencies, the τ measurements and their uncertainties are updated to account for the updated values of the external parameters. The dependencies on common systematic effects are also determined in size and sign, and all the common systematic dependencies of different measurements are used together with the published statistical and systematic uncertainties and correlations in order to compute an updated all-inclusive variance and covariance matrix of the experimental inputs of the fit.

The fit parameters correspond to all measured τ branching fractions and ratios, to some non-measured branching fractions and ratios like for instance $\mathcal{B}(\tau^- \to \pi^- K^0_L K^0_L \nu_\tau)$ and to one nuisance variable. When discussing the fit results in the following, the fit χ^2, the number of degrees of freedom, the residuals and pulls all refer to the subset of fit parameters that correspond to τ branching fractions and ratios, excluding nuisance variables. Due to the small number of nuisance fit parameters with respect to the tau decay fit parameters, we assume that the "restricted" fit χ^2, residuals and pulls approximately share the statistical properties expected for a minimum χ^2 fit. The fit parameters are optimized while respecting relations described by a series of constraint equations. All the experimental inputs and all the constraint equations are reported in the τ Listings section that follows this review. In some cases, constraints describe approximate relations that nevertheless hold within the present experimental precision. For instance, the constraint $\mathcal{B}(\tau^- \to K^-K^-K^+\nu_\tau) = \mathcal{B}(\tau^- \to K^-\phi\nu_\tau) \times \mathcal{B}(\phi \to K^+K^-)$ is only empirically justified within the current experimental evidence. The constraint equations between the τ branching fractions and ratios include quantities other than τ branching fractions and branching ratios, like for instance

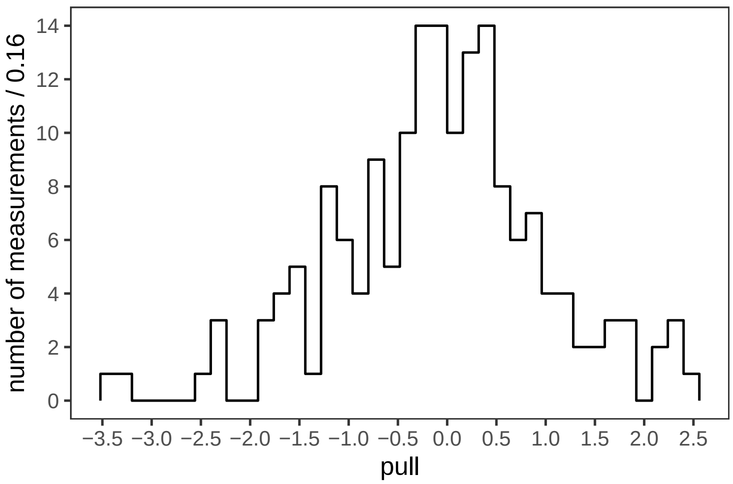

Figure 58.1: Pulls of individual measurements against the respective fitted quantity.

the branching fractions of the η and ω mesons. We neglect the uncertainties on these values for all quantities except $\mathcal{B}(a_1^- \to \pi^-\gamma)$, whose value and uncertainty were estimated by ALEPH [1] to be $(0.21 \pm 0.08)\cdot 10^{-2}$, relying on a measurement of $\Gamma(a_1^- \to \pi^-\gamma)$ [4]. This quantity is included in the fit parameters as nuisance variable, with a χ^2 term corresponding to its estimate, which is accomodated without a dedicated treatment in the general framework of the fit procedure described in Section 58.5. We assume that

$$\begin{aligned}\mathcal{B}(\tau^- \to a_1^-\nu_\tau) = {} & \mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\nu_\tau \text{ (ex.}K^0, \omega)) + \\ & \mathcal{B}(\tau^- \to \pi^- 2\pi^0\nu_\tau \text{ (ex.}K^0)) + \\ & \mathcal{B}(\tau^- \to a_1^-(\pi^-\gamma)\nu_\tau) \ ,\end{aligned} \tag{58.1}$$

neglecting the observed but negligible branching fractions to other modes, and that

$$\mathcal{B}(\tau^- \to a_1^-(\pi^-\gamma)\nu_\tau) = \mathcal{B}(\tau^- \to a_1^-\nu_\tau) \cdot \mathcal{B}(a_1^- \to \pi^-\gamma) \ . \tag{58.2}$$

The values of all other quantities in the constraint equations are taken from the 2024 edition of the Review of Particle Physics.

In the fit, uncertainty scale factors are applied to the published uncertainties of measurements only if significant inconsistency between different measurements remain after accounting for all relevant uncertainties and correlations. When performing the fit with no scale factors, the two measurements of $\mathcal{B}(\tau^- \to K^-K^-K^+\nu_\tau)$ have pulls exceeding $5\,\sigma$ from the fit values. There are 171 pulls, one per measurement. They are partially correlated, and the effective number of independent pulls is equal to the number of degrees of freedom of the fit, 126. The probability of getting pulls equal or larger than either one of the two very large pulls in a sample of 126 is smaller than the probability of a $3\,\sigma$ deviation for a Normal variable. Therefore, it has been decided to apply an uncertainty scale factor of 5.4 on all measurements of $\mathcal{B}(\tau^- \to K^-K^-K^+\nu_\tau)$ (one by BaBar and one by Belle). The scale factor has been computed according to the standard PDG procedure. After applying the scale factor, the pull distribution of the measurements in figure 58.1 is reasonably Normal and the pull probability distribution in figure 58.2 is reasonably flat.

Considering only the residuals with respect to τ branching fractions and ratios measurements, the constrained fit has a χ^2 of 138 for 126 degrees of freedom, corresponding to a χ^2 probability

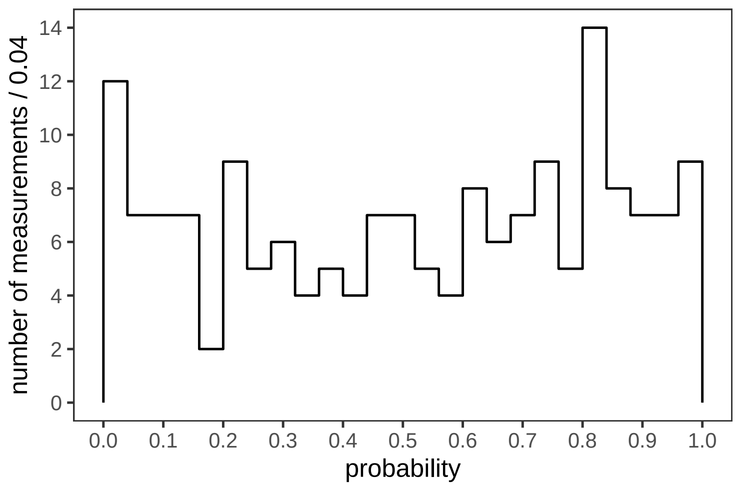

Figure 58.2: Probability of individual measurement pulls against the respective fitted quantity.

of 21.3%. We use 171 measurements and 84 constraints on the branching fractions and ratios to determine 129 quantities, consisting of 112 branching fractions and 17 branching ratios. The constraints include the unitarity constraint on the sum of all the exclusive τ decay modes, $\mathcal{B}_{\rm all} = 1$. If the unitarity constraint is released, the fit result for $\mathcal{B}_{\rm all}$ is consistent with unitarity with $1 - \mathcal{B}_{\rm all} = (0.07 \pm 0.11)\%$.

For the convenience of summarizing the fit results, we list in Table 58.1 the values and uncertainties for a set of 46 "basis" decay modes, from which all remaining branching fractions and ratios can be obtained using the constraints. The sum of all known τ branching fractions, which adds up to one according to the unitarity constraint, can be obtained by summing the fit values of all "basis" modes with a few adjustements:

- the modes $\mathcal{B}(\tau^- \to \pi^- K^0_S K^0_S \nu_\tau)$ and $\mathcal{B}(\tau^- \to \pi^- K^0_S K^0_S \pi^0 \nu_\tau)$ are both summed with a weight of 2, in order to include the corresponding modes with K_L's, which are predicted to have the same respective branching fractions by the constraint equations;
- the mode $\mathcal{B}(\tau^- \to K^- \phi \nu_\tau)$ has a weight smaller than 1 because some of its final states are described by other "basis" modes;
- the modes $\mathcal{B}(\tau^- \to \pi^- 2\pi^0 \nu_\tau\ ({\rm ex.}K^0))$ and $\mathcal{B}(\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau\ ({\rm ex.}K^0, \omega))$ have weights slighly larger than 1 to account for the $\mathcal{B}(\tau^- \to a_1^-(\pi^- \gamma) \nu_\tau)$ mode, which is predicted by them according to the constraint Eqs. 58.1 and 58.2.

Table 58.1 reports the weights with which the "basis" modes are summed to get the total τ branching fraction. The correlation matrix between the basis modes is reported in the τ Listings.

In defining the fit constraints and in selecting the modes that sum up to one we made some assumptions and choices. We assume that some channels, like $\tau^- \to \pi^- K^+ \pi^- \geq 0\pi^0 \nu_\tau$ and $\tau^- \to \pi^+ K^- K^- \geq 0\pi^0 \nu_\tau$, have negligible branching fractions as expected from the Standard Model, even if the experimental limits for these branching fractions are not very stringent. The 95% confidence level upper limits are $\mathcal{B}(\tau^- \to \pi^- K^+ \pi^- \geq 0\pi^0 \nu_\tau) < 0.25\%$ and $\mathcal{B}(\tau^- \to \pi^+ K^- K^- \geq 0\pi^0 \nu_\tau) < 0.09\%$, values not so different from measured branching fractions for allowed 3-prong modes containing charged kaons. For decays to final states containing one neutral kaon we assume that the branching fractions with the K^0_L are the same as the corresponding one with a K^0_S. On decays with two neutral kaons we assume that the branching fractions with $K^0_L K^0_L$ are the same as

Table 58.1: Values, uncertainties and unitarity-constraint coefficients of the 46 "basis" modes of the τ branching fractions and ratios fit.

decay mode	fit result (%)	coefficient
$\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$	0.1737 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$	0.1785 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to \pi^- \nu_\tau)$	0.1082 ± 0.0005	1.0000
$\mathcal{B}(\tau^- \to K^- \nu_\tau)$	0.00697 ± 0.00010	1.0000
$\mathcal{B}(\tau^- \to \pi^- \pi^0 \nu_\tau)$	0.2549 ± 0.0009	1.0000
$\mathcal{B}(\tau^- \to K^- \pi^0 \nu_\tau)$	0.00433 ± 0.00015	1.0000
$\mathcal{B}(\tau^- \to \pi^- 2\pi^0 \nu_\tau\ (\text{ex.}K^0))$	0.0926 ± 0.0010	1.0022
$\mathcal{B}(\tau^- \to K^- 2\pi^0 \nu_\tau\ (\text{ex.}K^0))$	$(0.65 \pm 0.22) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- 3\pi^0 \nu_\tau\ (\text{ex.}K^0))$	0.0104 ± 0.0007	1.0000
$\mathcal{B}(\tau^- \to K^- 3\pi^0 \nu_\tau\ (\text{ex.}K^0, \eta))$	$(0.48 \pm 0.21) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to h^- 4\pi^0 \nu_\tau\ (\text{ex.}K^0, \eta))$	0.0011 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to \pi^- \bar{K}^0 \nu_\tau)$	0.00838 ± 0.00014	1.0000
$\mathcal{B}(\tau^- \to K^- K^0 \nu_\tau)$	0.001486 ± 0.000034	1.0000
$\mathcal{B}(\tau^- \to \pi^- \bar{K}^0 \pi^0 \nu_\tau)$	0.00382 ± 0.00013	1.0000
$\mathcal{B}(\tau^- \to K^- K^0 \pi^0 \nu_\tau)$	0.00150 ± 0.00007	1.0000
$\mathcal{B}(\tau^- \to \pi^- \bar{K}^0 2\pi^0 \nu_\tau\ (\text{ex.}K^0))$	$(0.26 \pm 0.23) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- K^0_S K^0_S \nu_\tau)$	$(235 \pm 6) \cdot 10^{-6}$	2.0000
$\mathcal{B}(\tau^- \to \pi^- K^0_S K^0_L \nu_\tau)$	0.00108 ± 0.00024	1.0000
$\mathcal{B}(\tau^- \to \pi^- K^0_S K^0_S \pi^0 \nu_\tau)$	$(18.2 \pm 2.1) \cdot 10^{-6}$	2.0000
$\mathcal{B}(\tau^- \to \pi^- K^0_S K^0_L \pi^0 \nu_\tau)$	$(0.32 \pm 0.12) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \bar{K}^0 h^- h^- h^+ \nu_\tau)$	$(0.25 \pm 0.20) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau\ (\text{ex.}K^0, \omega))$	0.0899 ± 0.0005	1.0022
$\mathcal{B}(\tau^- \to \pi^- \pi^+ \pi^- \pi^0 \nu_\tau\ (\text{ex.}K^0, \omega))$	0.0274 ± 0.0007	1.0000
$\mathcal{B}(\tau^- \to h^- h^- h^+ 2\pi^0 \nu_\tau\ (\text{ex.}K^0, \omega, \eta))$	0.0010 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to \pi^- K^- K^+ \nu_\tau)$	0.001435 ± 0.000027	1.0000
$\mathcal{B}(\tau^- \to \pi^- K^- K^+ \pi^0 \nu_\tau)$	$(61 \pm 18) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- \pi^0 \eta \nu_\tau)$	0.00139 ± 0.00007	1.0000
$\mathcal{B}(\tau^- \to K^- \eta \nu_\tau)$	$(155 \pm 8) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to K^- \pi^0 \eta \nu_\tau)$	$(48 \pm 12) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- \bar{K}^0 \eta \nu_\tau)$	$(94 \pm 15) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- \pi^+ \pi^- \eta \nu_\tau\ (\text{ex.}K^0))$	$(220 \pm 13) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to K^- \omega \nu_\tau)$	$(0.41 \pm 0.09) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to h^- \pi^0 \omega \nu_\tau)$	0.0041 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to K^- \phi \nu_\tau)$	$(44 \pm 16) \cdot 10^{-6}$	0.8300
$\mathcal{B}(\tau^- \to \pi^- \omega \nu_\tau)$	0.0195 ± 0.0006	1.0000
$\mathcal{B}(\tau^- \to K^- \pi^- \pi^+ \nu_\tau\ (\text{ex.}K^0, \omega))$	0.00293 ± 0.00007	1.0000
$\mathcal{B}(\tau^- \to K^- \pi^- \pi^+ \pi^0 \nu_\tau\ (\text{ex.}K^0, \omega, \eta))$	$(0.39 \pm 0.14) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- 2\pi^0 \omega \nu_\tau)$	$(72 \pm 16) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to 2\pi^- \pi^+ 3\pi^0 \nu_\tau\ (\text{ex.}K^0, \eta, \omega, f_1))$	$(14 \pm 27) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to 3\pi^- 2\pi^+ \nu_\tau\ (\text{ex.}K^0, \omega, f_1))$	$(775 \pm 30) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to K^- 2\pi^- 2\pi^+ \nu_\tau\ (\text{ex.}K^0))$	$(0.6 \pm 1.2) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to 2\pi^- \pi^+ \omega \nu_\tau\ (\text{ex.}K^0))$	$(84 \pm 6) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to 3\pi^- 2\pi^+ \pi^0 \nu_\tau\ (\text{ex.}K^0, \eta, \omega, f_1))$	$(38 \pm 9) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau\ (\text{ex.}K^0))$	$(1.1 \pm 0.6) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- f_1 (2\pi^- 2\pi^+) \nu_\tau)$	$(52 \pm 4) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^- 2\pi^0 \eta \nu_\tau\ (\text{ex.}K^0))$	$(0.19 \pm 0.04) \cdot 10^{-3}$	1.0000

the ones with $K^0_S K^0_S$.

58.3 New measurements in this edition

With respect to the 2024 editions of the τ Listings and of this Review, a new precise measurement of the branching ratio $\mathcal{B}_\mu/\mathcal{B}_e = \mathcal{B}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)/\mathcal{B}(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$ by the Belle II collaboration [5] has been added. The updated fit has a slightly smaller value of $\mathcal{B}_\mu = \mathcal{B}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)$ and a slightly larger value of $\mathcal{B}_e = \mathcal{B}(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$, while $\mathcal{B}_\mu/\mathcal{B}_e$ has decreased and is closer to the Standard Model prediction. The other fit branching fractions and ratios have negligible changes.

58.4 Overconsistency of leptonic branching fraction and ratio measurements

As observed in the previous editions of this review, measurements of the leptonic branching fractions are more consistent with each other than expected from the quoted uncertainties on the individual measurements. When fitting all measurements of $\mathcal{B}_\mu$, $\mathcal{B}_e$ and $\mathcal{B}_\mu/\mathcal{B}_e$, the χ^2/n.d.o.f. is 2.6/11 and the probability of getting a smaller χ^2 is 0.53%.

58.5 Technical implementation of the fit

The fit computes a set of quantities denoted with q_i by minimizing a χ^2 while respecting a series of equality constraints on the q_i. The quantities q_i represent τ branching fractions and branching ratios, and nuisance variables. The fit minimization procedure is equivalent to choosing a set of basis fit variables, using functions of these basis variables to predict measurements, and determining these basis variables by minimizing the measurements' χ^2. The χ^2 is computed using the measurements m_i and their covariance matrix M_{ij} as $\chi^2 = (m_i - A_{ik}q_k)^t M_{ij}^{-1}(m_j - A_{jl}q_l)$, where the model matrix A_{ij} is used to get the vector of the predicted measurements m'_i from the vector of the fit parameters q_j as $m'_i = A_{ij}q_j$. There is one fit variable for each of the modes that have measurements, therefore $A_{ij} = 1$ when q_j corresponds to the τ branching fraction or branching ratio of the measurement m_i, and $A_{ij} = 0$ otherwise. The constraints are equations involving the fit parameters. The fit does not impose limitations on the functional form of the constraints. In summary, the fit requires:

$$\min\left[\chi^2(q_k)\right] = \min\left[(m_i - A_{ik}q_k)^t M_{ij}^{-1}(m_j - A_{jl}q_l)\right] , \quad (58.3)$$

$$\text{subjected to} \quad f_r(q_s) - c_r = 0 , \quad (58.4)$$

where the left term of Eq. 58.4 defines the constraint expressions. Using the method of Lagrange multipliers, a set of equations is obtained by taking the derivatives with respect to the fitted quantities q_k and the Lagrange multipliers λ_r of the sum of the χ^2 and the constraint expressions multiplied by the Lagrange multipliers λ_r, one for each constraint:

$$\begin{aligned} &\min\left[(m_i - A_{ik}q_k)^t M_{ij}^{-1}(m_j - A_{jl}q_l) + 2\lambda_r(f_r(q_s) - c_r)\right] = \\ &= \min\left[\tilde{\chi}^2(q_k, \lambda_r)\right] , \\ &(\partial/\partial q_k, \partial/\partial\lambda_r)\left[\tilde{\chi}^2(q_k, \lambda_r)\right] = 0 . \end{aligned} \quad (58.5)$$

Eq. 58.5 defines a set of equations for the vector of the unknowns (q_k, λ_r), some of which may be non-linear, in case of non-linear constraints. An iterative minimization procedure approximates at each step the non-linear constraint expressions by their first order Taylor expansion around the current values of the fitted quantities, $\bar{q}_s$:

$$f_r(q_s) - c_r = f_r(\bar{q}_s) + \left.\frac{\partial f_r(q_s)}{\partial q_s}\right|_{\bar{q}_s} (q_s - \bar{q}_s) - c_r ,$$

which can be written as

$$B_{rs}q_s - c'_r ,$$

where c'_r are the resulting known terms, independent of q_s at first order. After the linearization of the constraint expressions in $\tilde{\chi}^2(q_k, \lambda_r)$, the differentiation by q_k and λ_r is trivial and leads to a set of linear equations

$$A^t_{ki} M^{-1}_{ij} A_{jl} q_l + B^t_{kr} \lambda_r = A^t_{ki} M^{-1}_{ij} m_j \ , \tag{58.6}$$

$$B_{rs} q_s = c'_r \ , \tag{58.7}$$

which can be expressed as

$$F_{ij} u_j = v_i \ , \tag{58.8}$$

where $u_j = (q_k, \lambda_r)$ and v_i is the vector of the known constant terms running over the index k and then r in the right terms of Eq. 58.6 and Eq. 58.7, respectively. Solving the equation set in Eq. 58.8 by matrix inversion gives the the fitted quantities and their variance and covariance matrix, using the measurements and their variance and covariance matrix. The fit procedure starts by computing the linear approximation of the non-linear constraint expressions around the quantities seed values. With an iterative procedure, the unknowns are updated at each step by solving the equations and the equations are then linearized around the updated values, until the variation of the fitted unknowns is reduced below a numerically small threshold.

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